GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
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3. exercise sheet for Mathematics for advanced materials science

(first name) (last name)

(student id number)

3.1. *(Computing with complex exponential function)* (4 credits)

For real x, write the following complex number in the form $a + \mathrm{i}b$ with real numbers a and b.

$$\sum_{\substack{k=-3 \\ k\neq 0}}^{3} \frac{\mathrm{i}}{k} \exp(2\pi \mathrm{i} k x).$$

(Hint: the sum is over $k = \pm 1, \pm 2, \pm 3$ without $k = 0$. You should get some sines or cosines depending on x; the imaginary part b should look particularly simple.)

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 26.10.2021, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=3543`
`https://www.math.tugraz.at/~mtechnau/teaching/2021-w-mams.html`

3.2. *(Complex and real forms of Fourier series)* (4 credits)

Let $c_0, c_{\pm 1}, \dots, c_{\pm K}$ be complex numbers. Find complex numbers a_k and b_k such that for every real x

$$\sum_{k=-K}^{K} c_k \exp(ikx) = \frac{a_0}{2} + \sum_{k=1}^{K} (a_k \cos(kx) + b_k \sin(kx)).$$

$a_k =$ [ ] and $b_k =$ [ ].

3.3. *(Laplace transform)* (4 credits)

Let x be a solution to the following initial value problem:

$$\begin{cases} \text{differential equation: } 3\ddot{x} + x \overset{!}{=} \sin \text{ on } \mathbb{R}_+, \\ \text{initial conditions: } \begin{cases} \dot{x}(0) \overset{!}{=} 1, \\ x(0) \overset{!}{=} 1. \end{cases} \end{cases}$$

Find the Laplace transform $\mathscr{L}\{x\}$.

(Hint: you may use $\mathscr{L}\{\sin\}(s) = 1/(s^2+1)$. You may check your solution using $\mathscr{L}\{x\}(0) = 4$ and $\mathscr{L}\{x\}(2) \approx 0.70769$.)

$\mathscr{L}\{x\}(s) =$ [ ].

3.4. *(Laplace transform)* (4 credits)

Find $\mathscr{L}\{f\}$ where $f(t) = t\sin(t)\exp(t)$.

(Hint: $\mathscr{L}\{f\}(4) = 0.06$. To find the solution you can try to use partial integration a couple of times. If done correctly, four(!) partial integrations should suffice. Alternatively, you are free to use Proposition 2.4 and Table 1 from the lecture notes. This should be *much* easier.)

$\mathscr{L}\{f\}(s) =$ [ ].