GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
Marc Technau

Winter term 2021
Graz, 26.10.2021

4. exercise sheet for Mathematics for advanced materials science

4.1. *(Laplace transform)*
In exercise 3.3 you have computed

$$\mathscr{L}\{x\}(s) = \frac{3s(s^2+s+1)+4}{3s^4+4s^2+1}$$

for the solution x to the following initial value problem:

$$\begin{cases} \text{differential equation: } 3\ddot{x} + x \overset{!}{=} \sin \text{ on } \mathbb{R}_+, \\ \text{initial conditions: } \begin{cases} \dot{x}(0) \overset{!}{=} 1, \\ x(0) \overset{!}{=} 1. \end{cases} \end{cases}$$

(a) Invert the above Laplace transform to find an expression for x.

(b) Use a computer to plot the function x on the interval $[0, 100]$.

(c) Show that x remains bounded and provide as good an upper bound for x as you can, that is, find $C \in \mathbb{R}$ as small as possible such that for every $t \in \mathbb{R}$ one has $x(t) \leq C$.

4.2. *(Laplace transform)*
Find a function f with $\mathscr{L}\{f\}(s) = \dfrac{s-2}{s^2+4}$.

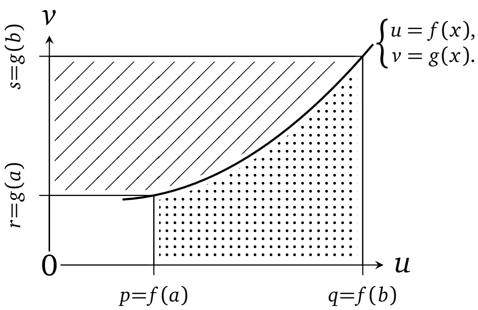
4.3. *(Proofs without words)*
Below you find visual proofs of two results.

(a) Describe what is being proved.

(b) Give an explanation of at least one of the proofs.

Adapted from Roger B. Nelsen's book *Proofs Without Words II*:

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 02.11.2021, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=3543`
`https://www.math.tugraz.at/~mtechnau/teaching/2021-w-mams.html`

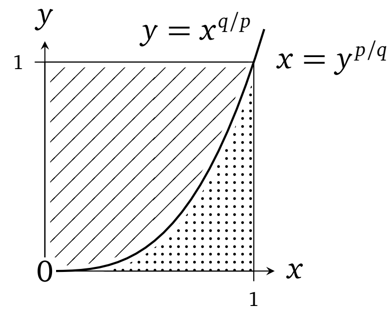


$$\text{Area}\ \boxed{/\!/} + \text{Area}\ \boxed{\vdots\vdots} = qs - pr,$$

$$\int_r^s u\,\mathrm{d}v + \int_p^q v\,\mathrm{d}u = uv\Big|_{(p,r)}^{(q,s)},$$

$$\int_a^b f(x)g'(x)\,\mathrm{d}x = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)\,\mathrm{d}x\,.$$

—Richard Courant



$$\int_0^1 \left(t^{p/q} + t^{q/p}\right)\mathrm{d}t = 1.$$

—Peter R. Newbury