GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
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Winter term 2021
Graz, 02.10.2021

5. exercise sheet for Mathematics for advanced materials science

(first name)	(last name)
(student id number)	

5.1. *(Solving a system of linear equations)* (4 credits)

Consider the following system of linear equations:

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 3 & 5 & 0 & 3 \\ 4 & 0 & 2 & 1 \\ 0 & 2 & 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 1 \\ 2 \\ 0 \\ 4 \end{pmatrix}.$$

Find the correct value of n such that the above system makes sense (i.e., such that the matrix–vector product on the left hand side can be computed). Subsequently determine all solutions to the above system.
(Hint: recall Gauß's algorithm from your "*Mathematik für ChemikerInnen 2*" course.)

$$n = \boxed{}, \quad \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}.$$

5.2. *(Solving a system of linear equations)* (4 credits)

Find all solutions $(x_1, x_2, x_3) \in \mathbb{R}^3$ to the following system of linear equations:

$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 09.11.2021, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=3543`
`https://www.math.tugraz.at/~mtechnau/teaching/2021-w-mams.html`

5.3. *(Finding a matrix representation)* (4 credits)

For each of the following linear maps f_ν, determine the matrix A_ν representing f_ν.

(a) $f_1 \colon \mathbb{R} \to \mathbb{R}$, $x \mapsto -4x$.

(b) $f_2 \colon \mathbb{R}^4 \to \mathbb{R}^2$, $\vec{x} \mapsto (x_1 - x_3, x_2)$.

(c) $f_3 \colon \mathbb{R}^4 \to \mathbb{R}^4$, $\vec{x} \mapsto (x_1 - x_3, x_2, x_1, x_1 + x_3)$.

(d) $f_4 \colon \mathbb{R}^4 \to \mathbb{R}^4$, $\vec{x} \mapsto \vec{y}$, where the vector $\vec{y}$ is determined from $\vec{x}$ such that the following equation is satisfied for all t

$$\frac{\mathrm{d}}{\mathrm{d}t}(x_1 + x_2 t + x_3 t^2 + x_4 t^3) = (y_1 + y_2 t + y_3 t^2 + y_4 t^3).$$

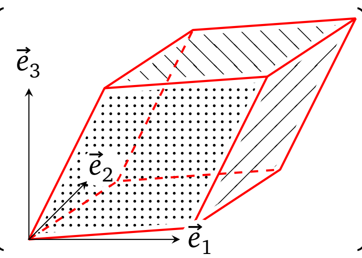
5.4. *(Volume of a parallelepiped)* (4 credits)

Compute the volume of the parallelepiped

$$\varparallelogram := \varparallelogram(\vec{v}, \vec{w}, \vec{z}) := \{\, \lambda_1 \vec{v} + \lambda_2 \vec{w} + \lambda_3 \vec{z} : 0 \le \lambda_1, \lambda_2, \lambda_3 \le 1 \,\}.$$

spanned by the vectors $\vec{v} = (1/5, 1, 0)$, $\vec{w} = (1, 1/5, 0)$ and $\vec{z} = (1/2, 0, 1)$.

volume($\varparallelogram$) = volume (



) = $\boxed{}$.

(Hint: use Cavalieri's principle which implies that volume($\varparallelogram(\vec{v}, \vec{w}, \vec{z})$) = volume($\varparallelogram(\vec{v} - \lambda\vec{w}, \vec{w}, \vec{z})$) for any $\lambda \in \mathbb{R}$. Similar formulae hold for subtracting a multiple of an argument of $\varparallelogram$ from another argument of $\varparallelogram$. Geometrically this can be thought of as replacing the initial parallelepiped with another parallelepiped that resembles a cuboid more closely, provided that λ is chosen suitably. The volume of a cuboid is easy to compute.)