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6. exercise sheet for Mathematics for advanced materials science

6.1. *(Laplace transform)*
Compute $\mathscr{L}\{t \mapsto e^{\mathrm{i}t}\}(s) = \int_0^\infty e^{\mathrm{i}t} e^{-st}\,\mathrm{d}t$ and compare real and imaginary parts to prove that

$$\mathscr{L}\{\cos\}(s) = \frac{s}{s^2+1} \quad \text{and} \quad \mathscr{L}\{\sin\}(s) = \frac{1}{s^2+1}.$$

6.2. *(Computing determinants)*
Compute the determinant of each of the following matrices:

(a) $\begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$,

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 0 \end{pmatrix}$,

(c) $\begin{pmatrix} \cos(\varphi)\sin(\theta) & r\cos(\varphi)\cos(\theta) & -r\sin(\varphi)\sin(\theta) \\ \sin(\varphi)\sin(\theta) & r\sin(\varphi)\cos(\theta) & r\cos(\varphi)\sin(\theta) \\ \cos(\theta) & -r\sin(\theta) & 0 \end{pmatrix}$ for $r, \varphi, \theta \in \mathbb{R}$.

(Hint: for (c) employ the identity $\cos(\varphi)^2 + \sin(\varphi)^2 = |\exp(\mathrm{i}\varphi)| = 1$ from Theorem 1.3. Your final result should only depend on r and θ and look very simple.)

6.3. *(Inverting matrices)*
Find the inverse matrix A^{-1} of

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 5 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

(Hint: there are several ways to do this. From your "*Mathematik für ChemikerInnen 2*" course you may know a variant of Gauß's algorithm which accomplishes this. Alternatively, you may solve the system of linear equations $A\vec{b}_j \overset{!}{=} \vec{e}_j$ for each $j = 1, 2, 3$. What do the vectors $\vec{b}_j$ have to do with A^{-1}? Lastly, you could also use Cramer's rule. You may verify the validity of your solution by checking that $AA^{-1} = \mathbf{1}_3$.)

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 16.11.2021, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=3543`
`https://www.math.tugraz.at/~mtechnau/teaching/2021-w-mams.html`