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8. exercise sheet for Mathematics for advanced materials science

8.1. *(Orthogonal matrices)*
A matrix $A \in \mathbb{R}^{n\times n}$ is called ***orthogonal*** if $A^{\mathrm{T}}A = \mathbf{1}_n$. For the sake of concreteness, suppose that $n = 3$ for the remainder of the exercise. Let $\vec{a}_{\bullet 1}, \vec{a}_{\bullet 2}, \vec{a}_{\bullet 3} \in \mathbb{R}^3$ denote the columns of A.

(a) Show that A is orthogonal if and only if

$$\|\vec{a}_{\bullet j}\| = 1 \quad \text{and} \quad \vec{a}_{\bullet j} \cdot \vec{a}_{\bullet k} = 0$$

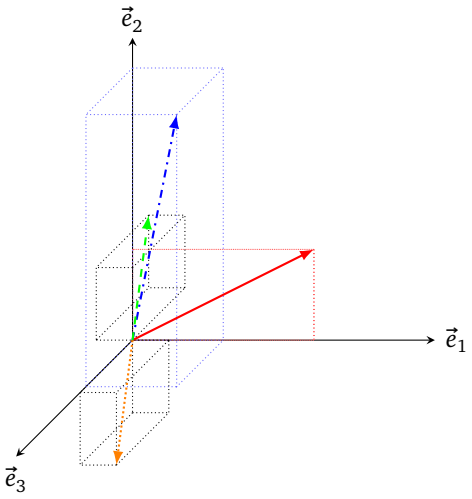
holds for all $1 \le j, k \le 3$, $j \ne k$.
(Hint: write $A = (a_{ij})_{i,j=1}^3$ and compute $A^{\mathrm{T}}A$. Now try to see some norms and dot products in there.)

(b) For each of the following three matrices, find values to put into the last column such that the resulting matrix is orthogonal if such values exist. If no such values exist, argue why this is the case.

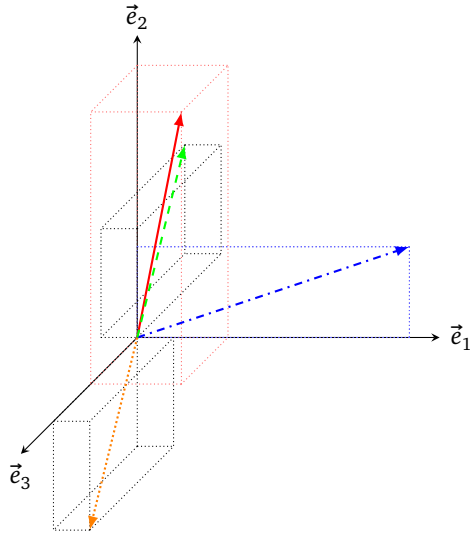
$$\begin{pmatrix} 1 & 0 & v_1 \\ 0 & -1 & v_2 \\ 0 & 0 & v_3 \end{pmatrix}, \quad \begin{pmatrix} 1/\sqrt{3} & 0 & w_1 \\ 1/\sqrt{3} & -1/\sqrt{2} & w_2 \\ 1/\sqrt{3} & 1/\sqrt{2} & w_3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & z_1 \\ 1 & -1 & z_2 \\ 1 & 1 & z_3 \end{pmatrix}.$$

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 02.12.2021, 12:15 o'clock. https://tc.tugraz.at/main/course/view.php?id=3543
https://www.math.tugraz.at/~mtechnau/teaching/2021-w-mams.html

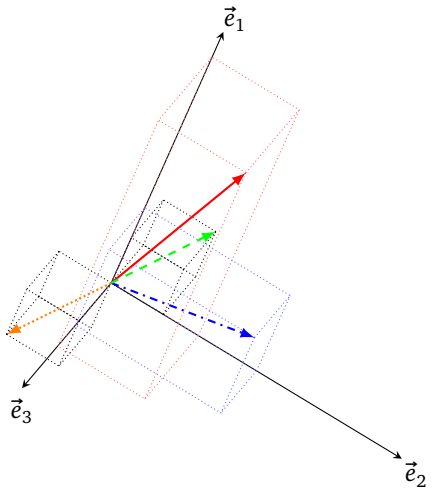
8.2. *(Cross products and orientation)*
In each of the figures below you see a vector $\vec{v}$ drawn as ⟶ and a vector $\vec{w}$ drawn as -·→. Discern for each figure whether the vector $\vec{v} \times \vec{w}$ is ⟶ or - - →.



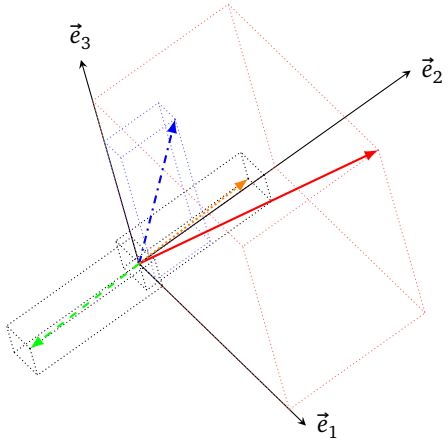
○ ⟶,
○ - - →,



○ ⟶,
○ - - →,



○ ⟶,
○ - - →,



○ ⟶,
○ - - →.

(Hint: pay very close attention to the direction of the three standard unit vectors $\vec{e}_1$, $\vec{e}_2$ and $\vec{e}_3$ for every figure separately.)