GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
Marc Technau

Winter term 2022
Graz, 25.10.2022

4. exercise sheet for Engineering Mathematics

(first name) (last name)

(student id number)

4.1. *(Solving a system of linear equations)* (4 credits)
Consider the following system of linear equations:

$$\begin{pmatrix} 4 & 0 & 2 & 1 \\ 1 & 0 & 2 & 0 \\ 3 & 5 & 0 & 3 \\ 0 & 2 & 5 & 0 \\ 4 & 5 & 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 4 \\ 3 \end{pmatrix}.$$

Find the correct value of n such that the above system makes sense (i.e., such that the matrix–vector product on the left hand side can be computed). Subsequently determine all solutions to the above system.

$$n = \square, \quad \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix}.$$

4.2. *(Solving a system of linear equations)* (4 credits)
Find *all* solutions $(x_1, x_2, x_3) \in \mathbb{R}^3$ to the following system of linear equations:

$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 3 \\ 3 \end{pmatrix}.$$

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 01.11.2022, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=4636`
`https://www.math.tugraz.at/~mtechnau/teaching/2022-w-engimaths.html`

4.3. *(Finding a matrix representation)* (4 credits)

For each of the following linear maps f_ν, determine the matrix A_ν representing f_ν.

(a) $f_1 : \mathbb{R} \to \mathbb{R}$, $x \mapsto -3x$.

(b) $f_2 : \mathbb{R}^4 \to \mathbb{R}^2$, $\vec{x} \mapsto (x_2 - x_1, x_3)$.

(c) $f_3 : \mathbb{R}^4 \to \mathbb{R}^4$, $\vec{x} \mapsto (x_1 - x_3, x_2, x_1, x_1 + x_3)$.

(d) $f_4 : \mathbb{R}^4 \to \mathbb{R}^4$, $\vec{x} \mapsto \vec{y}$, where the vector $\vec{y}$ is determined from $\vec{x}$ such that the following equation is satisfied for all t

$$\frac{\mathrm{d}}{\mathrm{d}t}(x_1 + x_2 t + x_3 t^2 + x_4 t^3) = y_1 + y_2 t + y_3 t^2 + y_4 t^3.$$

4.4. *(Composition of maps)* (4 credits)

Consider the linear maps

$$f : \mathbb{R}^3 \to \mathbb{R}^2,\ \vec{v} \mapsto \begin{pmatrix} v_1 + 2v_2 + v_3 \\ 2v_2 + v_3 \end{pmatrix}, \quad \text{and} \quad g : \mathbb{R}^2 \to \mathbb{R}^3,\ \vec{w} \mapsto \begin{pmatrix} w_1 - w_2 \\ w_2/2 \\ 0 \end{pmatrix}.$$

Compute the following:

(a) $(f \circ g)(\vec{w}) = \begin{pmatrix} \square \\ \square \end{pmatrix}$, $(g \circ f)(\vec{v}) = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}$,

(b) the matrices A, B, C, D representing f, g, $f \circ g$ and $g \circ f$ respectively,

(c) the matrices AB and BA.