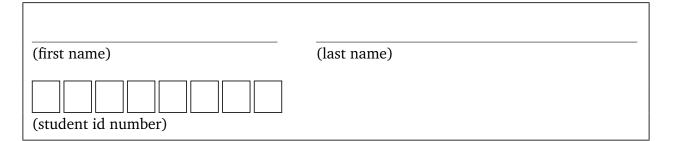


4. exercise sheet for Engineering Mathematics



4.1. *(Solving a system of linear equations)* Consider the following system of linear equations:

(4	0	2	1)			(0)	
1	0	2	0	(x_1)		1	
3	5	0	3 0		!	2	
0	2	5	0	$\left(x_{n} \right)$		4	
\4	5	2	3)			\3/	

Find the correct value of n such that the above system makes sense (i.e., such that the matrix–vector product on the left hand side can be computed). Subsequently determine all solutions to the above system.

$$n =$$
 , $\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ \end{pmatrix}.$

4.2. (Solving a system of linear equations) (4 credits) Find *all* solutions $(x_1, x_2, x_3) \in \mathbb{R}^3$ to the following system of linear equations:

$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 3 \\ 3 \end{pmatrix}.$$

(4 credits)

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 01.11.2022, 23:55 o'clock. https://tc.tugraz.at/main/course/view.php?id=4636 https://www.math.tugraz.at/~mtechnau/teaching/2022-w-engimaths.html

4.3. (Finding a matrix representation)

(4 credits)

For each of the following linear maps f_{ν} , determine the matrix A_{ν} representing f_{ν} .

- (a) $f_1: \mathbb{R} \to \mathbb{R}, x \mapsto -3x$.
- (b) $f_2 \colon \mathbb{R}^4 \to \mathbb{R}^2, \ \vec{x} \mapsto (x_2 x_1, x_3).$
- (c) $f_3: \mathbb{R}^4 \to \mathbb{R}^4, \vec{x} \mapsto (x_1 x_3, x_2, x_1, x_1 + x_3).$
- (d) $f_4: \mathbb{R}^4 \to \mathbb{R}^4, \vec{x} \mapsto \vec{y}$, where the vector \vec{y} is determined from \vec{x} such that the following equation is satisfied for all t

$$\frac{\mathrm{d}}{\mathrm{d}t}(x_1 + x_2t + x_3t^2 + x_4t^3) = y_1 + y_2t + y_3t^2 + y_4t^3.$$

4.4. *(Composition of maps)* Consider the linear maps

(4 credits)

$$f: \mathbb{R}^3 \to \mathbb{R}^2, \ \vec{v} \mapsto \begin{pmatrix} v_1 + 2v_2 + v_3 \\ 2v_2 + v_3 \end{pmatrix}, \quad \text{and} \quad g: \mathbb{R}^2 \to \mathbb{R}^3, \ \vec{w} \mapsto \begin{pmatrix} w_1 - w_2 \\ w_2/2 \\ 0 \end{pmatrix}.$$

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Compute the following:

(a)
$$(f \circ g)(\vec{w}) = \left(\boxed{\qquad} \right), (g \circ f)(\vec{v}) = \left(\boxed{\qquad} \right),$$

(b) the matrices A, B, C, D representing f, g, $f \circ g$ and $g \circ f$ respectively,

(c) the matrices *AB* and *BA*.