GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
Marc Technau

Winter term 2022
Graz, 06.12.2022

10. exercise sheet for Engineering Mathematics

(first name)	(last name)
(student id number)	

10.1. *(Differentiation)* (4 credits)

Consider the two maps $f:\mathbb{R}^2\to\mathbb{R}^2$, $(x,y)\mapsto(xy,x-y)$, and $g:\mathbb{R}^2\to\mathbb{R}$, $(v,w)\mapsto v^2+w^2$. Compute the following:

(a) $(g\circ f)(x,y)$;

(b) the Jacobian matrices $J_f(x,y)$, $J_g(v,w)$, and $J_{g\circ f}(x,y)$,

(c) the matrix–matrix product $J_g(f(x,y))J_f(x,y)$.

10.2. *(Gradient)* (4 credits)

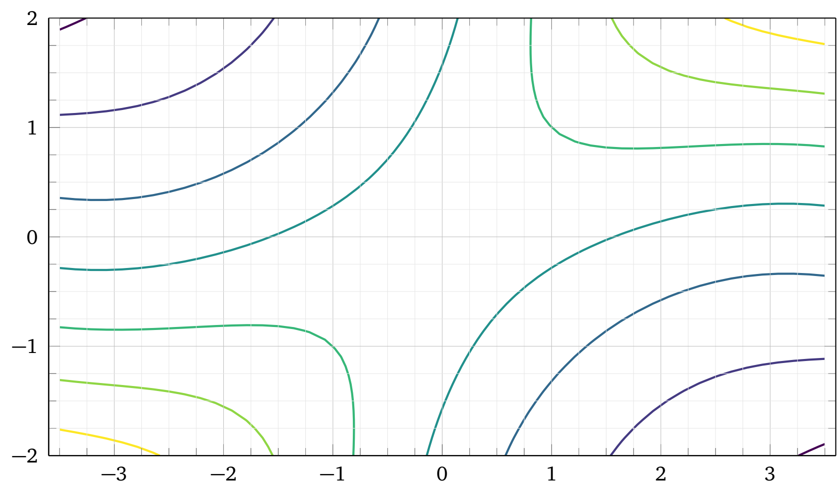
Consider the map $f:\mathbb{R}^2\to\mathbb{R}$, $(x,y)\mapsto\cos(x-y)+xy$.

(a) Compute $J_f(x,y)$.

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 13.12.2022, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=4636`
`https://www.math.tugraz.at/~mtechnau/teaching/2022-w-engimaths.html`

(b) Compute $\operatorname{grad} f(x,y)$.

(c) Pick three distinct points $(x,y) \in [-3,3] \times [-2,2]$ for which you compute the gradient $\operatorname{grad} f(x,y)$ numerically and draw it as a vector based at (x,y) in the following picture:



(Hint: the curved lines are curves on which f is constant.)

10.3. *(Potentials)* (4 credits)

(a) Find a function $f : \mathbb{R}^2 \to \mathbb{R}$ with $\operatorname{grad} f(x,y) = (2xy-1, x^2)$.

(b) Find a function $g : \mathbb{R}^2 \to \mathbb{R}$ with $\operatorname{grad} g(x,y) = (\sin(x-y)+x\cos(x-y), -x\cos(x-y))$.

(Hint: expand the definition of the gradient and see what this tells you about the function f or g you need to find. Once you have f or g, it is also easy to check that your solution is correct; just compute the gradient.)

10.4. *(Divergence)* (4 credits)

Let $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2$, $(x,y) \mapsto (F_1(x,y), F_2(x,y))$ be a vector field. Define the ***divergence*** $\operatorname{div} \vec{F}(x,y)$ of $\vec{F}$ at (x,y) to be $\partial_1 F_1(x,y) + \partial_2 F_2(x,y)$ if the appearing partial derivatives exist. Compute $\operatorname{div} \operatorname{grad} f(x,y)$ and $\operatorname{div} \operatorname{grad} g(x,y)$, where f and g are the functions from exercise 10.3. (Hint: here the main task is to decypher the notation.)