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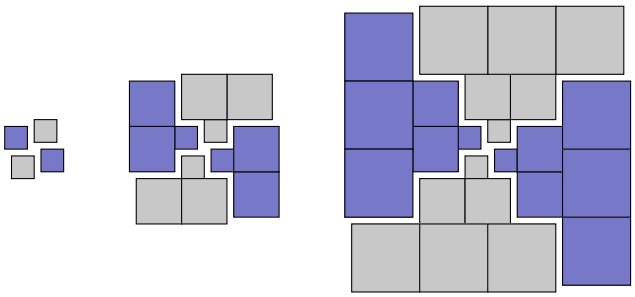
# 13. exercise sheet for Engineering Mathematics

**13.1.** *(Sums and integrals)*
This exercise is aimed at getting a feel for the definition of the Darboux integral (see Chapter 7 of the lecture notes).

(a) Stare at the picture below and use this to find a formula for $\sum_{k=1}^{n} k^3 = 1^3+2^3+\ldots+n^3$.



(b) Using a partition of $[0,1]$ into $n$ pieces of equal length, the definition of the Darboux integral yields

$$\sum_{k=1}^{n}((k-1)/n)^3(1/n) \le \int_0^1 x^3\,\mathrm{d}x \le \sum_{k=1}^{n}(k/n)^3(1/n).$$

Use your answer from (a) to compute the outer two sums. Also compute the integral in the middle and show that, as $n \to \infty$, both sums converge to that integral (this corresponds to taking the partition to be much finer).

**13.2.** *(A variant of the Gaussian integral)*
Compute $\int_{-\infty}^{\infty} \exp(-x^2+x)\,\mathrm{d}x$.
(Hint: remember the 'completing the square' trick from your school days and use the result from Example 7.3.)

**13.3.** *(Volume of a solid)*
Compute the volume of

$$R = \{(x,y,z) \in \mathbb{R}^3 : 0 \le x \le 1,\ 0 \le y \le x,\ 0 \le z \le xy\}.$$

(Hint: compute $\int_R 1\,\mathrm{d}^3\vec{x}$ using Fubini's theorem.)