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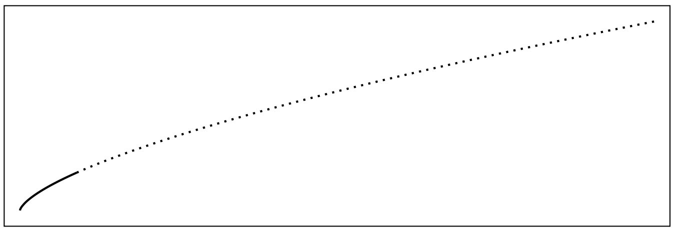
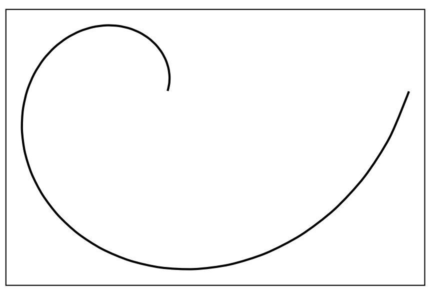
15. exercise sheet for Engineering Mathematics

15.1. *(Length of a curve, II)*
Determine the length of the curve $\{(\frac{3}{2}(t-1)^{3/2}, t) \in \mathbb{R}^2 : 1 \le t \le 2\}$.



(Hint: your final answer should be approximately equal to 2.74264.)

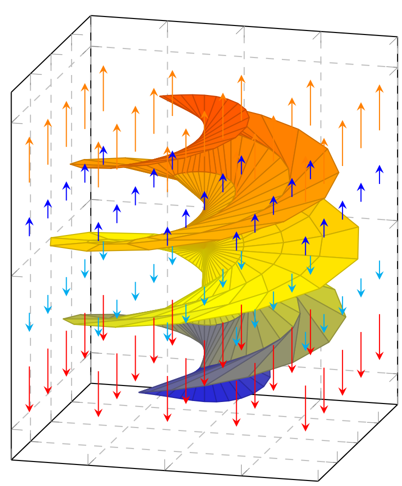
15.2. *(Length of a curve, III)*
Determine the length of the curve $\{(\cos(\pi t)\exp(t), \sin(\pi t)\exp(t)) \in \mathbb{R}^2 : 0 \le t \le 2\}$.



(Hint: your final answer should be approximately equal to 21.06413.)

15.3. *(Surface integrals and line integrals)*
Let $U = \{(r, \varphi) \in \mathbb{R}^2 : r^2 + \varphi^2 \le 1^2\}$ be the closed disk in $\mathbb{R}^2$ with centre $(0,0)$ and radius 1 and consider the surface $S = \vec{\Phi}(U)$, where

$$\vec{\Phi} : U \to \mathbb{R}^3, \quad \begin{pmatrix} r \\ \varphi \end{pmatrix} \mapsto \begin{pmatrix} r\cos(2\pi\varphi) \\ r\sin(2\pi\varphi) \\ \varphi \end{pmatrix}.$$



Consider also the map

$$\vec{\gamma} : [0, 2\pi] \to \mathbb{R}^3, \quad t \mapsto \vec{\Phi}(\cos(t), \sin(t)),$$

which parametrises the curve $C = \vec{\gamma}([0, 2\pi])$.

Moreover, consider the vector field $\vec{K}: \mathbb{R}^3 \to \mathbb{R}^3$ given by $\vec{K}(x, y, z) = (0, 0, z)$.
Compute:

(a) $\vec{n}(r, \varphi) := \dfrac{\partial \vec{\Phi}}{\partial r}(r, \varphi) \times \dfrac{\partial \vec{\Phi}}{\partial \varphi}(r, \varphi)$,

(b) $\operatorname{rot} \vec{K}(x, y, z)$,

(c) $\displaystyle\iint_S \vec{K} \cdot \mathrm{d}\vec{A}$,

(d) $\displaystyle\int_C \operatorname{rot} \vec{K} \cdot \mathrm{d}\vec{s}$.

(Hint: for the definition of the integrals in (c) and (d), see § 7.3 of the lecture notes. The computations in (a), (b), and (d) should be rather straight-forward. The computation in (c) requires more work, also because you need to parametrise U somehow. Use polar coordinates as defined in § 6.4.)