GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
Marc Technau

Winter term 2022
Graz, 13.10.2022

2. exercise sheet for Mathematics for Advanced Materials Science

(first name)	(last name)

(student id number)

2.1. *(Linear ordinary differential equations)* (4 credits)

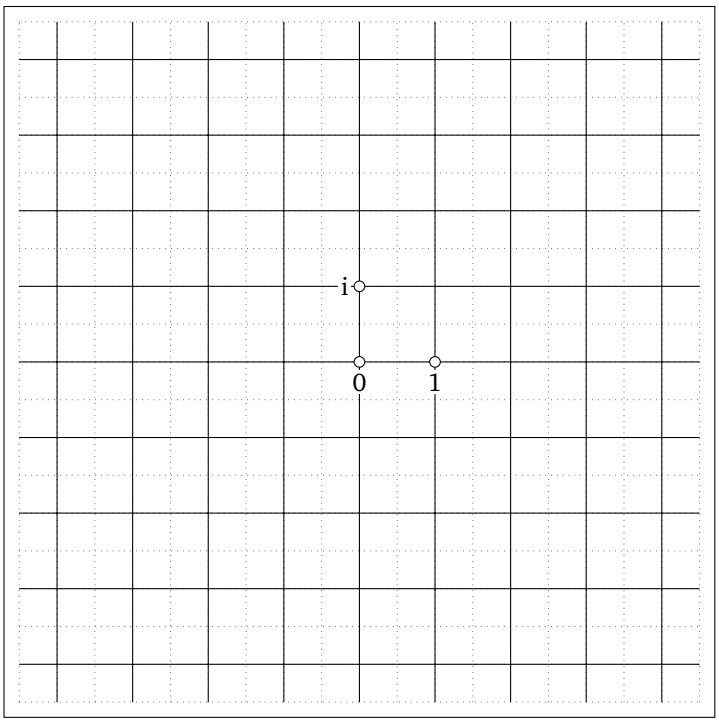
(a) Let λ_0 be an arbitrary number. Verify that both $t \mapsto \exp(\lambda_0 t)$ and $t \mapsto t\exp(\lambda_0 t)$ satisfy the differential equation $\ddot{x} - 2\lambda_0\dot{x} + \lambda_0^2 x \stackrel{!}{=} 0$.

(b) Find a solution x to the differential equation from (a) with $x(0) = 2$ and $\dot{x}(0) = 3$.

(For part (a) of this exercise you may need more space than is given here. Use a separate sheet.)

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 20.10.2022, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=3543`
`https://www.math.tugraz.at/~mtechnau/teaching/2022-w-mams.html`

2.2. *(Working with complex numbers)* (4 credits)
Sketch the following set of complex numbers below: $\{z \in \mathbb{C} : 3 \leq |z| < 4,\ \mathrm{Re}(z) \geq -2\}$.



2.3. *(Solving quadratic equations)* (4 credits)
Find all (complex) solutions to the equation $X^2 - 6X + 12 \stackrel{!}{=} 0$.
(Hint: you can use the formula for finding roots of quadratic polynomials and $\sqrt{-1} = \pm\mathrm{i}$.)

2.4. *(Complex differentiation)* (4 credits)
Let z be a complex number. Compute:

(a) $\dfrac{\mathrm{d}}{\mathrm{d}z}\big(z^7 + 4z^2 - \cos(z) + 42\sin(z)\big) =$,

(b) $\dfrac{\mathrm{d}}{\mathrm{d}z}\left(\exp\left(\dfrac{z^3}{z+1}\right)(1+z)^2\right) =$ for $z \neq -1$.

(Hint: just pretend that z is a real number and differentiate as you would have done in school. $\exp' = \exp$, $\cos' = -\sin$, $\sin' = \cos$.)