GRAZ UNIVERSITY OF TECHNOLOGY
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4. exercise sheet for Mathematics for Advanced Materials Science

(first name)	(last name)
(student id number)	

4.1. *(Laplace transform)* (4 credits)
Find $\mathscr{L}\{f\}$ where $f(t) = t\sin(t)\exp(t)$.

$$\mathscr{L}\{f\}(s) = \boxed{}\,.$$

(Hint: $\mathscr{L}\{f\}(4) = 0.06$. To find the solution you can try to use integration by parts a couple of times. If done correctly, integrating by parts four times should suffice. Alternatively, you are free to use Proposition 2.4 and Table 1 from the lecture notes.)

4.2. *(Laplace transform)* (4 credits)
In exercise 3.4 you have computed

$$\mathscr{L}\{x\}(s) = \frac{3s(s^2+s+1)+4}{3s^4+4s^2+1}$$

for the solution x to the following initial value problem:

$$\begin{cases} \text{differential equation: } 3\ddot{x} + x \stackrel{!}{=} \sin \text{ on } \mathbb{R}_+, \\ \text{initial conditions: } \begin{cases} \dot{x}(0) \stackrel{!}{=} 1, \\ x(0) \stackrel{!}{=} 1. \end{cases} \end{cases}$$

Please submit your solutions digitally at the corresponding TeachCenter course. The deadline is 03.11.2022, 23:55 o'clock. `https://tc.tugraz.at/main/course/view.php?id=3543`
`https://www.math.tugraz.at/~mtechnau/teaching/2022-w-mams.html`

Invert the above Laplace transform to find an expression for x.

$$x(t) = \boxed{}\,.$$

(Hint: you can use $x(1) \approx 1.8352$ and $x(2) \approx 2.3259$ to verify your result.)

4.3. *(Laplace transform)* (4 credits)
Find a function f with $\mathscr{L}\{f\}(s) = \dfrac{s-2}{s^2+4}$.

$$f(t) = \boxed{}\,.$$

(Hint: you can use $f(1) \approx 0.10316$ and $f(\pi) = 1$ to verify your result.)

4.4. *(Laplace transform)* (4 credits)
Compute

$$\mathscr{L}\{t \mapsto e^{\mathrm{i}t}\}(s) = \int_0^\infty e^{\mathrm{i}t} e^{-st}\,\mathrm{d}t$$

and *use this* to deduce the following formulae:

(a) $\mathscr{L}\{\cos\}(s) = \dfrac{s}{s^2+1}$, and

(b) $\mathscr{L}\{\sin\}(s) = \dfrac{1}{s^2+1}$.

(Remark: unlike the other exercises above, this one actually asks for the computation that takes you to the final result. If you run out of space here, please use a separate sheet.)