GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
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2. exercise sheet for Engineering Mathematics

(first name) (last name)

(student id number)

2.1. *(Differentiation)* (4 credits)

Compute the following derivatives:

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2}{x^2} =$ [ ],

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2-5}{x^2+1} =$ [ ],

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\sin(\cos(x)^2) =$ [ ],

(d) $\frac{\mathrm{d}}{\mathrm{d}x}\arcsin(x) =$ [ ].

(Hint: For the third part, use that $\cos' = -\sin$ and $\sin' = \cos$. Moreover, "arcsin" is the inverse function of the sine function restricted to $(-\frac{\pi}{2}, \frac{\pi}{2})$. For computing its derivative, see how the derivative of arctan is computed at the end of § 0.6 of the lecture notes.)

2.2. *(Integration)* (4 credits)

Compute the following integrals:

(a) $\int_1^3 \left(x + \frac{1}{x} + \frac{1}{x^2}\right) \mathrm{d}x =$ [ ],

(b) $\int_{-1}^1 x\sin(x^2)\,\mathrm{d}x =$ [ ],

Please submit your solutions during the next lecture (18.10.2023).
https://www.math.tugraz.at/~mtechnau/teaching/2023-w-engimaths.html

(c) $\int_0^1 x^2 \exp(x)\,\mathrm{d}x =$ ______ .

(Please give *exact* values, and not approximations. For instance, do *not* write 0.6931 for log(2).)

Hint: All of the above exercises can be solved using the fundamental theorem of calculus. For (c) one would usually use a trick called "integration by parts"; see § 0.7.4 in the lecture notes If you do not know this trick, try to find $A, B, C \in \mathbb{R}$ such that $\frac{\mathrm{d}}{\mathrm{d}x}((A + Bx + Cx^2)\exp(x)) = x^2 \exp(x)$ and then apply the fundamental theorem.

2.3. *(Bessel's differential equation)* (4 credits)
Suppose that $y\colon \mathbb{R} \to \mathbb{R}$ is a non-zero solution to the differential equation

$$x^2 y''(x) + x y'(x) + x^2 y(x) \overset{!}{=} 0 \quad (\text{for all } x \in \mathbb{R}).$$

Suppose further that y can be written as a power series $y(x) = \sum_{n=0}^{\infty} a_n x^n$ with suitable coefficients $a_0, a_1, \ldots$. For the following tasks, please submit your solution on a separate sheet and *justify your computations*.

(a) Work as in exercise 1.3 to derive a formula for a_n, $n = 1, 2, 3, \ldots$. (Hint: "formula" is perhaps somewhat vague. Anyway, at the end you should be able to see that $a_1 = 0$ and $a_2 = 64^{-1} a_0$, for instance.)

(b) Suppose that $a_0 = 4$ and consider the polynomial $y_8(x) = \sum_{n=0}^{8} a_n x^n$. (This polynomial approximates y for small x, but we will not make this precise.) Compute $y_8(1)$. (Hint: if you can verify that $y_8(2) = 43/48$, then your answer for $y_8(1)$ is likely correct. Moreover, please give the exact answer as a fraction, and not just a decimal approximation.)

(c) Below one can see a plot of y, y_2, y_4 and y_6 . Use a computer to generate a plot of y_8 on the interval $[0, 5]$ and sketch it in the figure below or attach a printout of that plot.