GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
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Winter term 2023
Graz, 25.10.2023

4. exercise sheet for Engineering Mathematics

(first name) (last name)

(student id number)

4.1. *(Solving a system of linear equations)* (4 credits)
Make sure that you are familiar with the material presented in § 3.6 of the lecture notes. Consider the following system of linear equations:

$$\begin{pmatrix} 4 & 0 & 1 & 0 \\ 3 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 0 \\ 4 & 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}.$$

Find the correct value of n such that the above system makes sense (i.e., such that the matrix–vector product on the left hand side can be computed). Subsequently determine all solutions to the above system.

$$n = \boxed{}, \quad \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}.$$

4.2. *(Solving a system of linear equations)* (4 credits)
Find *all* solutions $(x_1, x_2, x_3) \in \mathbb{R}^3$ to the following system of linear equations:

$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & 5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} 5 \\ 5 \end{pmatrix}.$$

Please submit your solutions during the next lecture (08.11.2023).
https://www.math.tugraz.at/~mtechnau/teaching/2023-w-engimaths.html

4.3. *(Finding a matrix representation)* (4 credits)

For each of the following linear maps f_ν, determine the matrix A_ν representing f_ν.

(a) $f_1 \colon \mathbb{R} \to \mathbb{R}$, $x \mapsto -5x$.

(b) $f_2 \colon \mathbb{R}^4 \to \mathbb{R}^2$, $\vec{x} \mapsto (x_2 - x_3, x_3)$.

(c) $f_3 \colon \mathbb{R}^4 \to \mathbb{R}^4$, $\vec{x} \mapsto (x_1 + 5x_2 - x_3, x_2, x_1, x_2 + x_3)$.

(d) $f_4 \colon \mathbb{R}^4 \to \mathbb{R}$, $\vec{x} \mapsto \displaystyle\int_0^1 (x_1 + x_2 t + x_3 t^2 + x_4 t^3)\,\mathrm{d}t$.

(Hint: if your matrix A_4 contains a t in one of its entries, then it is wrong.)

4.4. *(Composition of maps)* (4 credits)

Consider the linear maps

$$f : \mathbb{R}^3 \to \mathbb{R}^2,\ \vec{v} \mapsto \begin{pmatrix} -v_2 + v_3 \\ v_1 - v_2 + v_3 \end{pmatrix}, \quad \text{and} \quad g : \mathbb{R}^2 \to \mathbb{R}^3,\ \vec{w} \mapsto \begin{pmatrix} w_2 - w_1 \\ -w_1 \\ 0 \end{pmatrix}.$$

Compute the following:

(a) $(f \circ g)(\vec{w}) = \begin{pmatrix} \boxed{} \\ \boxed{} \end{pmatrix}$, $(g \circ f)(\vec{v}) = \begin{pmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \end{pmatrix}$,

(b) the matrices A, B, C, D representing f, g, $f \circ g$ and $g \circ f$ respectively,

(c) the matrices AB and BA.

(Hint: reading § 3.1.3 of the lecture notes may be helpful but is not strictly necessary.)