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5. exercise sheet for Engineering Mathematics

5.1. *(Inverting matrices)*
Find the inverse matrix A^{-1} of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix}.$$

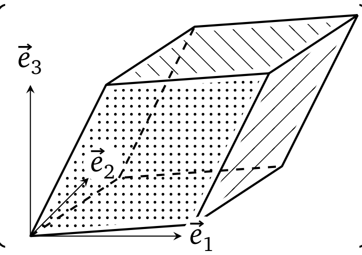
(Hint: there are several ways of doing this. For example, you may use the Gauß–Jordan algorithm from § 3.6.5 of the lecture notes. Alternatively, you may use Cramer's rule, Proposition 3.2.)

5.2. *(Volume of a parallelepiped)*
Compute the volume of the parallelepiped

$$\varnothing := \varnothing(\vec{v}, \vec{w}, \vec{z}) := \{\, \lambda_1 \vec{v} + \lambda_2 \vec{w} + \lambda_3 \vec{z} : 0 \leq \lambda_1, \lambda_2, \lambda_3 \leq 1 \,\}.$$

spanned by the vectors $\vec{v} = (1/5, 1, 0)$, $\vec{w} = (1, 1/5, 0)$ and $\vec{z} = (1/2, 0, 1)$.

volume(▱) = volume (



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(Hint: you can use Cavalieri's principle, or you can simply compute the determinant of an appropriate 3×3-matrix.)

5.3. *(Computing determinants)*
Compute the determinant of each of the following matrices:

(a) $\begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$,

(b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 0 \end{pmatrix}$,

(c) $\begin{pmatrix} \cos(\varphi)\sin(\theta) & r\cos(\varphi)\cos(\theta) & -r\sin(\varphi)\sin(\theta) \\ \sin(\varphi)\sin(\theta) & r\sin(\varphi)\cos(\theta) & r\cos(\varphi)\sin(\theta) \\ \cos(\theta) & -r\sin(\theta) & 0 \end{pmatrix}$ for $r, \varphi, \theta \in \mathbb{R}$.

(Hint: for (c), employ the identity $\cos(\varphi)^2 + \sin(\varphi)^2 = 1$. Your final result should only depend on r and θ and look very simple.)