GRAZ UNIVERSITY OF TECHNOLOGY
INSTITUTE OF ANALYSIS AND NUMBER THEORY
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Winter term 2023
Graz, 12.10.2023

2. exercise sheet for Mathematics for Advanced Materials Science

(first name) (last name)

(student id number)

2.1. *(Linear ordinary differential equations)* (4 credits)

(a) Let λ_0 be an arbitrary number. Verify that both $t \mapsto \exp(\lambda_0 t)$ and $t \mapsto t\exp(\lambda_0 t)$ satisfy the differential equation $\ddot{x} - 2\lambda_0\dot{x} + \lambda_0^2 x \stackrel{!}{=} 0$.

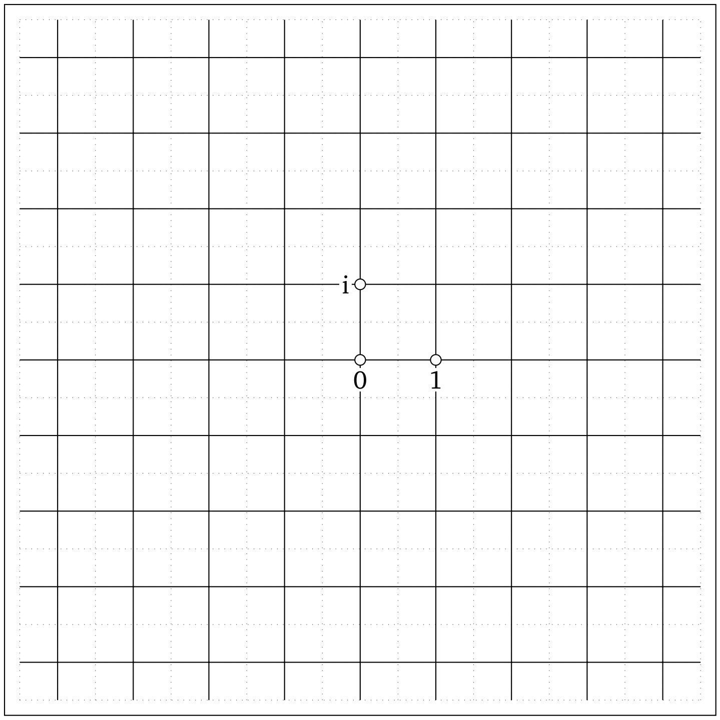
(b) Find a solution x to the differential equation from (a) with $x(0) = 3$ and $\dot{x}(0) = 2$.

(For part (a) of this exercise you may need more space than is given here. Use a separate sheet.)

Please submit your solutions during the next lecture (19.10.2023).
https://www.math.tugraz.at/~mtechnau/teaching/2023-w-mams.html

2.2. *(Working with complex numbers)* (4 credits)

Sketch the following set of complex numbers below: $\{ z \in \mathbb{C} : 2 \leq |z| < 4,\ \mathrm{Re}(z) \geq -2 \}$.



2.3. *(Solving quadratic equations)* (4 credits)

Find all (complex) solutions to the equation $X^2 - 2X + 12 \overset{!}{=} 0$.

(Hint: you can use the formula for finding roots of quadratic polynomials and $\sqrt{-1} = \pm \mathrm{i}$.)

2.4. *(Complex differentiation)* (4 credits)

Let z be a complex number. Compute:

(a) $\dfrac{\mathrm{d}}{\mathrm{d}z}\big(z^7 + 4z^2 - \cos(z^2) + 42\exp(z)\big) = \boxed{\qquad\qquad\qquad\qquad}$,

(b) $\dfrac{\mathrm{d}}{\mathrm{d}z}\left(\exp\left(\dfrac{z^2}{z+1}\right)(1+z)^2\right) = \boxed{\qquad\qquad\qquad\qquad}$ for $z \neq -1$.

(Hint: just pretend that z is a real number and differentiate as you would have done in that case. $\exp' = \exp$, $\cos' = -\sin$.)