



Diskrete Stochastik und Informationstheorie Exercise sheet $1 - \frac{14}{3}/2013$

Sigma algebras and some probability

Exercise 1 A dice takes values in $\Omega = \{1, 2, 3, 4, 5, 6\}$.

- a) What is the minimal σ -algebra that contains the information if the value of the dice is even or odd?
- b) What is the minimal σ -algebra that contains the information if the value of the dice is either in $\{1, 2\}$, in $\{3, 4\}$ or in $\{5, 6\}$?
- c) What is the σ -algebra that allows you to determine the value of the dice?

Exercise 2 Is $\{\emptyset, \{a\}, \{b, c\}, \{d, a\}, \{a, b, c, d\}\}$ a σ -algebra over $\{a, b, c, d\}$? If not, which elements are missing? What is the "information" described by this σ -algebra?

Exercise 3 Let F be an arbitrary family of subsets of a set X. Show that there exists a unique smallest σ -algebra which contains every set in F.

Exercise 4 Stefan has two kids, one of them is a girl. What is the chance that the other is a girl? Milka has two kids, the first born is a boy. What is the chance that the second is a girl?

Exercise 5 A fair dice is rolled first. It gives a random number k from $\{1, 2, 3, 4, 5, 6\}$. Next a fair coin is tossed k times. Assume, we know that all coins show heads, what is the probability that the score of the dice was equal to 5?

Exercise 6 Let X and Z be two discrete, independent rvs on $(\Omega, \mathcal{A}, \mathbb{P})$ taking only finitely many values in \mathbb{N} .

- a) Let $f : \mathbb{N} \to \mathbb{N}$ be a function. Show that Y = f(X) is again a rv.
- b) Show that (X, Z) is a rv.