TU Graz
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Department of Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie
Exercise sheet 10 – 13/6/2013

Data compression and codes

Exercise 40 *(Relative entropy is the cost of miscoding) Let the rv X take values in $\{1,2,3,4,5\}$. Consider two possible distributions p and q of this rv X, as well as two binary encodings C_1 and C_2 of it:*

x	$p(x)$	$q(x)$	$C_1(x)$	$C_2(x)$
1	$\frac{1}{2}$	$\frac{1}{2}$	0	0
2	$\frac{1}{4}$	$\frac{1}{8}$	10	100
3	$\frac{1}{8}$	$\frac{1}{8}$	110	101
4	$\frac{1}{16}$	$\frac{1}{8}$	1110	110
5	$\frac{1}{16}$	$\frac{1}{8}$	1111	111

1) *Calculate $H(p)$ and show that C_1 is optimal for p. Same for q and C_2.*

2) *Calculate $D(p||q)$ and quantify the loss when encoding with C_2 under p. Same for $D(q||p)$ and encoding with C_1 under q.*

Exercise 41 *Find the word lengths of an optimal binary encoding of $p = (0.01, \ldots, 0.01)$.*

Exercise 42 *We consider the discrete channel $Y = X + Z \mod 11$, where $X \in \mathcal{X} = \{0,1,\ldots,10\}$ and Z is a random noise with*

$$\mathbb{P}[Z=1] = \mathbb{P}[Z=2] = \mathbb{P}[Z=3] = \frac{1}{3}$$

such that Z and X are independent. Determine the capacity and the probability distribution on $\mathcal{X}$ that maximizes $I(X;Y)$.

Exercise 43 *Let $\mathcal{X} = \mathcal{Y} = \{0,1\}$. The transition probabilities are given by :*

$$p(0|0) = 1,\ p(1|0) = 0,\ p(0|1) = p(1|1) = \frac{1}{2}.$$

Determine the capacity and the probability distribution that maximizes $I(X;Y)$.