

Diskrete Stochastik und Informationstheorie

Exercise sheet 11 – 20/6/2013

Channels and capacity

Exercise 44 We consider two independent discrete channels $(\mathcal{X}_1, \mathcal{P}_1, \mathcal{Y}_1)$ and $(\mathcal{X}_2, \mathcal{P}_2, \mathcal{Y}_2)$. We can construct a new channel such that $x_1 \in \mathcal{X}_1$ and $x_2 \in \mathcal{X}_2$ will be transmitted in parallel (at the same time): so x_1 is mapped to some $y_1 \in \mathcal{Y}_1$ and x_2 is mapped to some $y_2 \in \mathcal{Y}_2$. Calculate the capacity of this joint channel.

Exercise 45 Let X and Y be two random variables with joint distribution:

$Y \setminus X$	0	1
a	$\frac{1}{2}$	$\frac{1}{6}$
b	$\frac{1}{12}$	$\frac{1}{4}$

From a realization of (x, y) of the random variables (X, Y) only the value of y can be observed and we want to give a “good” estimation of the value of x . In other words, we want to find a function $\hat{X}(y)$: with probability $p_i^{(y)}$ an observed symbol $y \in \{a, b\}$ will lead to the estimation $\hat{X}(y) = i$, for $i \in \{0, 1\}$. Find an estimator $\hat{X}(Y)$ that minimizes the probability of error $\mathbb{P}[\hat{X}(Y) \neq X]$.

Exercise 46 Let X and Y be two random variables with joint distribution

$Y \setminus X$	a	b	c
1	$\frac{1}{6}$	$\frac{1}{12}$	0
2	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$
3	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$

We are in the situation that the realization of Y are known while those of X are unknown. The function $\hat{X}(Y)$ denotes an estimator for X based on the observation of Y .

- 1) Find an estimator $\hat{X}(Y)$ that minimizes $\mathbb{P}[\hat{X}(Y) \neq X]$.
- 2) Compare the above result with the bound that you obtain using Fano’s inequality.

Exercise 47 We wish to encode a sequence of independent random variables with Bernoulli(α)-distribution over a binary symmetric channel with crossover probability p , i.e., $\mathbb{P}[X = 1, Y = 0] = \mathbb{P}[X = 0, Y = 1] = p$. Find conditions on α and p so that the probability of error goes to zero as n goes to infinity.