Graz University of Technology

Department of Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie
Exercise sheet 2 – 21/3/2013

Random variables

Exercise 7 *Let $X \sim Unif(\{1,2\})$ and $Y \sim Bin(2,1/2)$, and X and Y independent.*

a) *Describe the distribution of $Z = X + Y$ and its σ-algebra. It suffices to enumerate the atoms (subsets of $\{1,2\} \times \{0,1,2\}$) generating the σ-algebra.*

b) *Same for $W = Y - 2X$.*

Exercise 8 *The joint distribution of the vector (X,Y) is given by the following matrix:*

(i,j)	-1	2	3
0	0.03	0.16	0.12
1	0.07	0.35	0.27

For example, $\mathbb{P}((X,Y) = (0,-1)) = 0.03$.

a) *Calculate the marginal distributions of X and Y.*

b) *Are X and Y independent?*

Exercise 9 *Let X, Y be two independent, dice-valued (that means $Unif(\{1,\dots,6\})$) rvs.*

a) *Calculate $\mathbb{E}[2X + Y^2]$.*

b) *Calculate the covariance $Cov(X,Y) := \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$.*

c) *Are $X + Y$ and $X - Y$ independent?*

d) *Calculate the covariance of $(X + Y)$ and $(X - Y)$.*

Exercise 10 *Let X be a geometric distributed random variable on $\mathbb{N}$, i.e., $\mathbb{P}(X = k) = (1-p)^{k-1}p$ for some $p \in [0,1]$. Define the sequence of rvs $(X_n)_{n\geq 0}$ by*

$$X_n := \begin{cases} (1-p)^{1-n} & \textit{if } X = n \\ 0 & \textit{otherwise.} \end{cases}$$

a) *Does (X_n) converge in probability?*

b) *What can you say about the almost sure limit of X_n?*

c) *Can you interchange limit and expectation?*