TU Graz
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Department of Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie
Exercise sheet 4 – 25/4/2013

Relative entropy and mutual information

Exercise 16 *Let X and Y be two random variables with the following joint distribution:*

$Y \setminus X$	0	1
0	$\frac{1}{4}$	0
1	$\frac{3}{8}$	$\frac{3}{8}$

1) *Calculate the relative entropy of X and Y.*
2) *Calculate the joint information $I(X;Y)$.*
3) *The random variable Z is defined by*

$$Z = \begin{cases} 0, & \text{if } X + Y \leq 1 \\ 1, & \text{if } X + Y > 1. \end{cases}$$

Calculate $I(X;Y|Z)$.

Exercise 17 *Give, in each case, an example of random variables X, Y and Z such that*

$$(a)\ I(X;Y|Z) < I(X;Y) \qquad (b)\ I(X;Y|Z) > I(X;Y)$$

Exercise 18 *Show that a Markov triplet $X \to Y \to Z$ verifies*

$$\forall\ x, y, z : \quad \mathbb{P}[X = x|Y = y]\mathbb{P}[Z = z|Y = y] = \mathbb{P}[X = x, Z = z|Y = y]\,.$$

Exercise 19 *Prove the "log-sum-inequality".*

Exercise 20 *A function $\rho(x, y)$ is a metric on $\mathcal{P}$ if for all $x, y, z \in \mathcal{P}$:*

1) $\rho(x, y) \geq 0$,
2) $\rho(x, y) = \rho(y, x)$,
3) $\rho(x, y) = 0$ *if and only if* $x = y$,
4) $\rho(x, y) + \rho(y, z) \geq \rho(x, z)$.

Is the function $\rho(X, Y) = H(X|Y) + H(Y|X)$ a metric on the space of all discrete random variables? If not, how can you "turn" it into a metric ?