Graz University of Technology

Department of Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie
Exercise sheet 7 – 23/5/2013

Entropy rate and AEP

Exercise 29 *Let $X_1, X_2, \ldots$ be i.i.d. random variables with distribution given by $p(x), x \in \{1, \ldots, m\}$. Let $q(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} q(x_i)$, where q is a probability function on $\{1, 2, \ldots, m\}$ different of p.*

1) *Evaluate $\lim_{n\to\infty} \frac{1}{n} \log q(X_1, X_2, \ldots, X_n)$.*

2) *What is the limit of the log likelihood ratio $\frac{1}{n} \log \frac{q(X_1, X_2, \ldots, X_n)}{p(X_1, X_2, \ldots, X_n)}$? Interpret the result.*

Exercise 30 *A discrete source emits a sequence of binary bits that follow a stationary Markov chain. The Markov chain is described by the transition matrix*

$$P = \begin{pmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{pmatrix}$$

1) *Calculate the entropy rate of X_n.*

2) *How many 1's will you typically observe in a sequence of 100 digits.*

3) *Give bounds on the size of the typical sets.*

4) *Use Markov's inequality to bound the probability of observing a source sequence that contains more than k 1's. Let $\epsilon = 0.001$ and use the above bound to find the value k such that the probability of observing more than k times a 1 is less than ϵ.*

Exercise 31 (ruin problem) *Imagine you play black jack with a friend and you win 1€ with probability one half and you loose 1€ with probability one half. At the beginning you have n€ and your friend has m€. You play until the first player is broke.*

1) *Formulate a Markov chain that models the game.*

2) *Calculate the probability that you win. (You may consider the expectation of your money during the game.)*

3) *Comment on the statement "Never play again somebody who is richer".*