Graz University of Technology

Department of Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie
Exercise sheet 8 – 30/5/2013

Data compression

Exercise 32 *Let $\mathcal{X} = \{a, b, c, d\}$. The elements of $\mathcal{X}$ are encoded as follows:*

$$C(a) = 10, \quad C(b) = 00, \quad C(c) = 11, \quad C(d) = 110\,.$$

Check if the code C is nonsingular resp. instantaneous (prefix code).

Exercise 33 *Let $\mathcal{X} = \{a, b, c, d, e\}$. Construct a prefix code $C : \mathcal{X} \to \{0, 1\}^*$, so that*

$$l(C(a)) = 1, \quad l(C(b)) = 3, \quad l(C(c)) = 4, \quad l(C(d)) = 4, \quad l(C(e)) = 5\,.$$

Exercise 34 *An instantaneous code has word lengths $l_1, \ldots, l_m$, which satisfy the strict inequality*

$$\sum_{i=1}^{m} D^{-l_i} < 1\,.$$

The code alphabet is $\mathcal{D} = \{0, \ldots, D-1\}$. Show that there exist arbitrarily long sequences of code symbols in $\mathcal{D}^\star$ which can not be decoded into sequences of codewords.

Exercise 35 *The source coding theorem shows that the optimal code for a rv X has an expected length less than $H(X)+1$. Give an example of a rv for which the expected length of the optimal code is close to $H(X) + 1$. That is, construct for every $\varepsilon > 0$ a rv X_ε for which the optimal code fulfills $L_\varepsilon > H(X_\varepsilon) + 1 - \varepsilon$.*