TU Graz
Graz University of Technology

Department of
Mathematical Structure Theory

Diskrete Stochastik und Informationstheorie
Exercise sheet 9 – 6/6/2013

Data compression

Exercise 36 *Find a binary Huffman-Code of the distribution* $(\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{2}{15}, \frac{2}{15})$. *Argue that this code is also optimal for the distribution* $(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5})$.

Exercise 37 *Which of the following codes can not arise as Huffman-Codes?*

a) $\{01, 10\}$

b) $\{0, 10, 11\}$

c) $\{00, 01, 10, 110\}$

Exercise 38 *Let* $\mathcal{X} = \{a, b, c, d, e, f, g, h, i\}$. *The elements of* $\mathcal{X}$ *appear independently with the following probabilities:*

x	a	b	c	d	e	f	g	h	i
$p(x)$	0.3	0.2	0.15	0.1	0.05	0.05	0.08	0.04	0.03

Find the Huffman-Code and the expected code length of a letter in the

1) binary case,

2) ternary case.

Exercise 39 *Consider the homogeneous Markov chain* $X = (X_n)_{n\in\mathbb{N}}$ *on the state space* $S = \{s_1, s_2, s_3\}$ *with transition matrix*

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

1) Design three binary encodings C_1, C_2 *and* C_3 *of* S *such that the Markov process* X *can be sent with maximal compression by the following algorithm:*

- *Note the current symbol* $X_n = s_i$.
- *Select code* C_i.
- *Note the next symbol* $X_{n+1} = s_j$ *and sent the codeword in* C_i *corresponding to* s_j.
- *Repeat for the next symbol.*

2) What is the average message length of the next symbol conditioned on $X_n = s_i$*?*

3) What is the unconditional average number of bits per source symbol? Relate this to the entropy of the Markov chain.