Explicit Formulæ for some Numerical Triangles of Z. W. Trzaska

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In [3], the following modified numerical triangle Mnt1 is considered. The polynomials $T_k(q)$ are given by $T_0(q)=1$, $T_1(q)=1+q$ and the recursion formula

$$T_{k+1}(q)=(2+q)T_k(q)-T_{k-1}(q)\ . \tag{1}$$

The numbers $A(k,m)$ of the modified numerical triangle Mnt1 are defined as

$$A(k,m)=[q^m]T_k(q)\ .$$

(Read: "coefficient of q^m in $T_k(q)$.")

Theorem 1.

$$A(k,m)=\binom{k+m}{2m}\ .$$

The second modified numerical triangle Mnt2 is the following. The polynomials $P_k(q)$ are given by $P_0(q)=0$, $P_1(q)=1$ and the recursion formula

$$P_{k+1}(q)=(2+q)P_k(q)-P_{k-1}(q)\ .$$

The numbers $B(k,m)$ of the modified numerical triangle Mnt2 are defined as

$$B(k,m)=[q^m]P_k(q)\ .$$

Theorem 2.

$$B(k,m)=\binom{k+m}{2m+1}\ .$$

The paper gives a backward pointer to [2], which also has Mnt1, but also another one, $g(k,m)$, which is a little bit clumsy to describe, but for which I found the formula

Theorem 3.

$$g(k,m)=\frac{k^2-k+m^2}{m(2m-1)}\binom{k+m-2}{2m-2}\ .$$

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The paper [1] is also referenced, but it does not give more triangles than the ones just described.

We only give a proof of Theorem 1, the other ones being similar. The formula is evidently true for $m = 0$ and also for $k = 0$ and $k = 1$. Now, proceeding by induction and reading off the coefficient of q^m in equation (1) we get for $m, k \geq 1$

$$A(k+1, m) = 2A(k, m) + A(k, m-1) - A(k-1, m) \ .$$

It is a routine matter to check that $A(k, m) = \binom{k+m}{2m}$ fulfills this.

References

[1] Z. W. Trzaska. Numerical triangle, Fibonacci sequence and ladder networks: some further results. *Appl. Math. Lett.*, 6.4:55–61, 1993.

[2] Z. W. Trzaska. Modified numerical triangle and the Fibonacci sequence. *The Fibonacci Quarterly*, 32:124–129, 1994.

[3] Z. W. Trzaska. On Fibonacci hyperbolic trigonometry and modified numerical triangles. *The Fibonacci Quarterly*, 34:129–138, 1996.

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