Aggregation Models on Comb Lattices

(joint work with Wilfried Huss)

Ecaterina Sava

Graz University of Technology

La Pietra week in Probability, Florence, June 13-17, 2011

Internal Diffusion Limited Aggregation

- Random growth model.
- Let G be some infinite graph, and o some fixed vertex.
- one by one, particles perform discrete-time random walks.
- each particle starts from o and moves until it reaches a site unoccupied previously, where it stops.
- get a random subset of n occupied sites in G: internal DLA cluster $A(n)$ $\rightarrow$ the resulting random cluster of occupied sites after the nth particle stops.

Internal Diffusion Limited Aggregation

- Random growth model.
- Let G be some infinite graph, and o some fixed vertex.
- one by one, particles perform discrete-time random walks.
- each particle starts from o and moves until it reaches a site unoccupied previously, where it stops.
- get a random subset of n occupied sites in G: internal DLA cluster $A(n)$ $\rightarrow$ the resulting random cluster of occupied sites after the nth particle stops.

  Growth rule: Let $A(0) = \{o\}$ and define

  $$A(n+1) = A(n) \cup \{X^n(\tau_n)\},$$

  where $X^1, X^2, \ldots$ are independent random walks starting at o, and $\tau_n = \min\{t : X^n(t) \notin A(n)\}$.

Internal Diffusion Limited Aggregation

Graz University of Technology

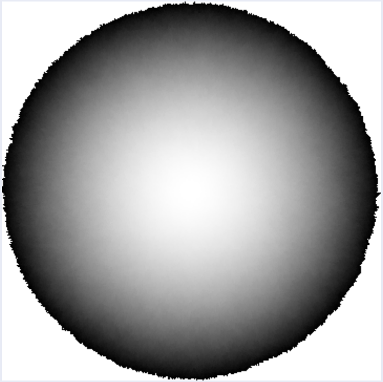
- Random growth model.
- Let G be some infinite graph, and o some fixed vertex.
- one by one, particles perform discrete-time random walks.
- each particle starts from o and moves until it reaches a site unoccupied previously, where it stops.
- get a random subset of n occupied sites in G: internal DLA cluster $A(n)$ → the resulting random cluster of occupied sites after the nth particle stops.

  Growth rule: Let $A(0) = \{o\}$ and define

$$A(n+1) = A(n) \cup \{X^n(\tau_n)\},$$

  where $X^1, X^2, \ldots$ are independent random walks starting at o, and $\tau_n = \min\{t : X^n(t) \notin A(n)\}$.
- Main question: limiting shape of $A(n)$ as $n \to \infty$?

The cluster A(n), n=100000



Theorem (Lawler-Bramson-Griffeath '92)

For simple random walks on $\mathbb{Z}^d$*,* $d \geq 2$*, the limiting shape of internal DLA is a ball:* $\forall \epsilon > 0$*, with probability* 1*:* $B_{r(1-\epsilon)} \subset A(\pi r^2) \subset B_{r(1+\epsilon)}$.

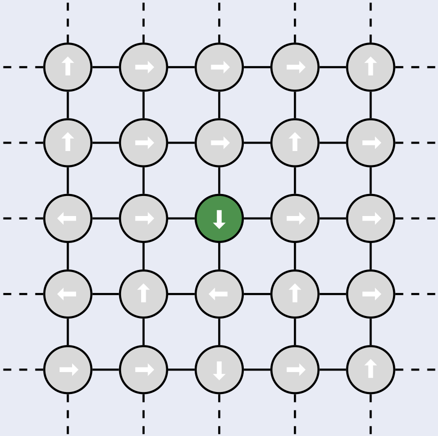
The Rotor-Router Walks

- The Rotor-Router is a deterministic analogue of random walk.
- For each vertex $v \in G$, we define a cyclical ordering of its neighbours.
- At each vertex we have a rotor (an arrow pointing to one of the neighbours).

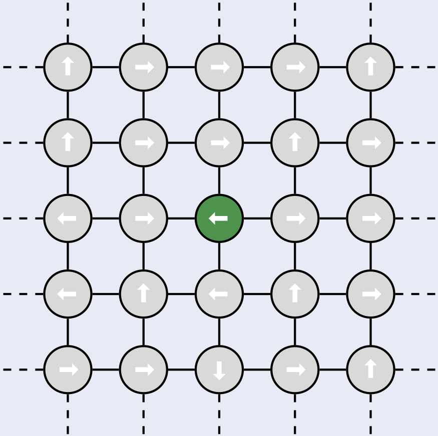
Transition rule: If a particle is at a vertex v

1. It first changes the rotor at v to point to its next neighbour,
2. then it moves into the direction of the rotor.

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence

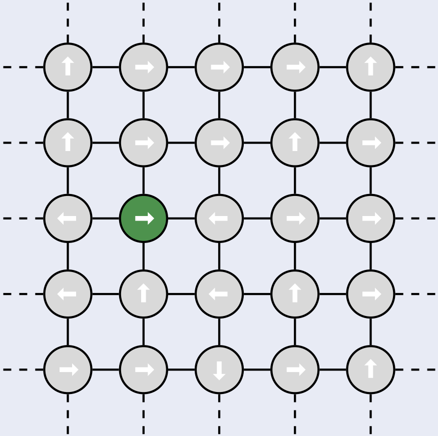


Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



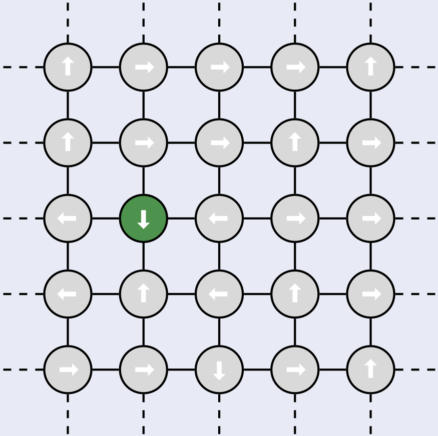
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



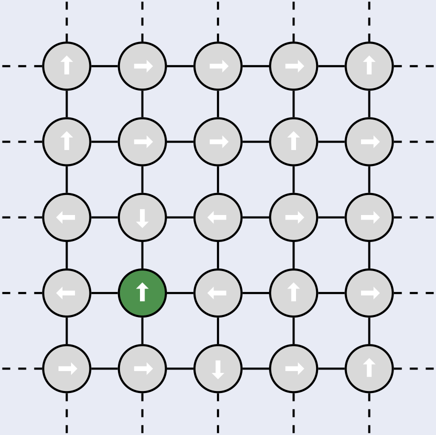
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



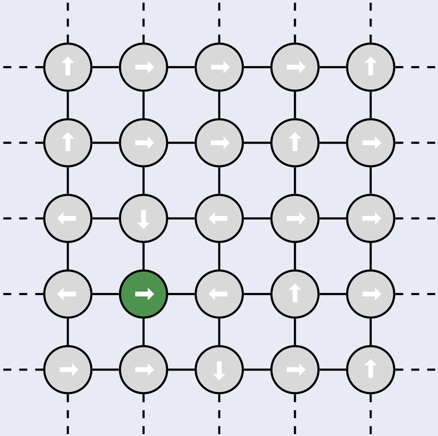
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



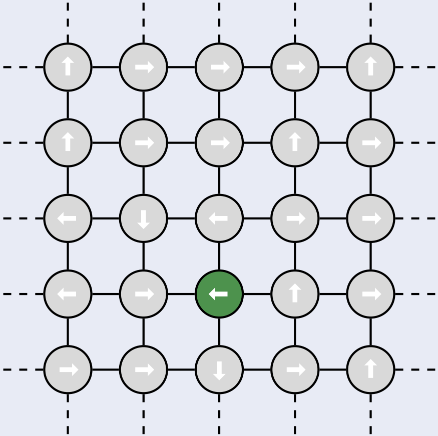
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



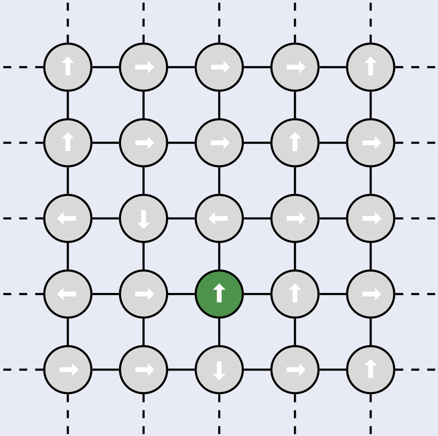
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



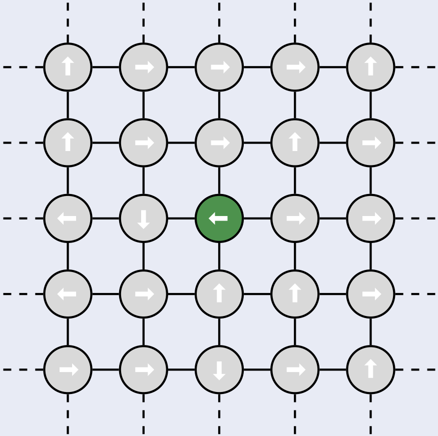
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



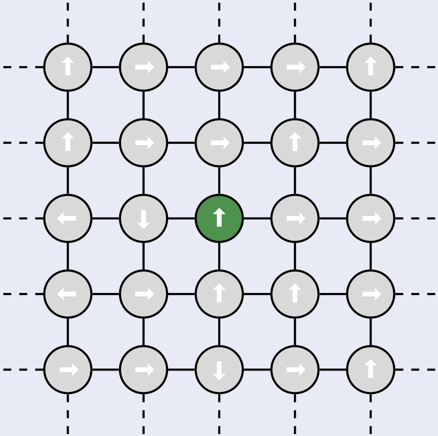
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



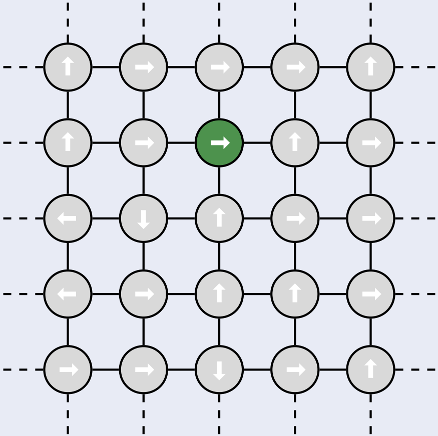
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



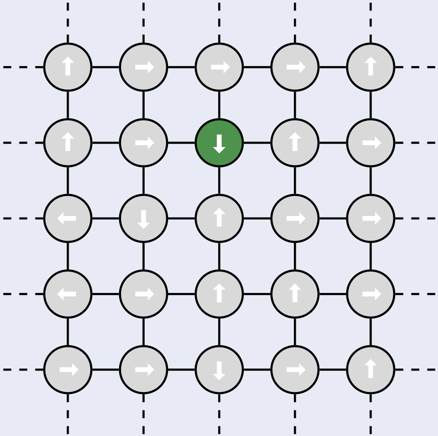
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



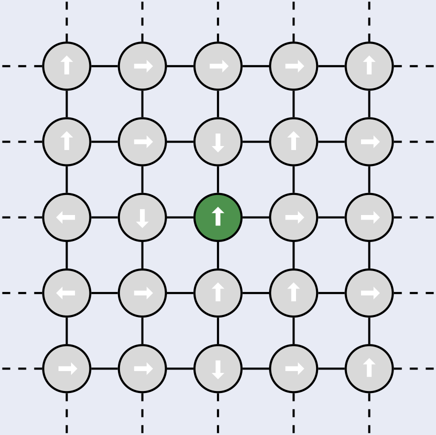
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



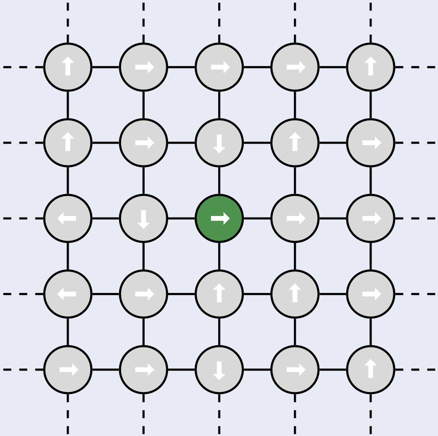
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



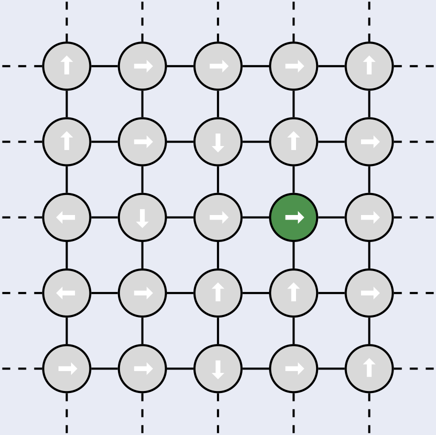
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

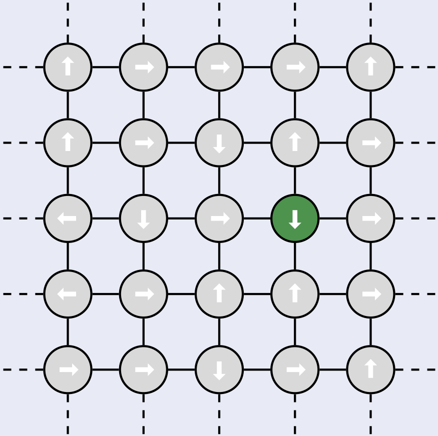
Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

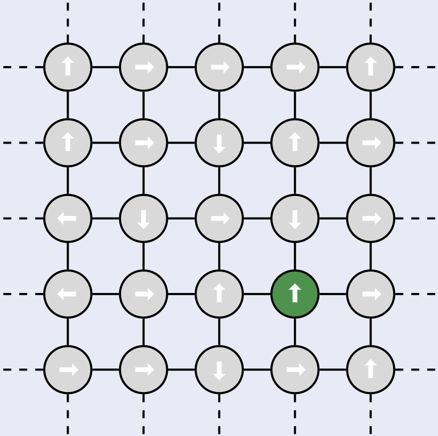
Example: $G = \mathbb{Z}^2$, clockwise rotor sequence

TU Graz
Graz University of Technology



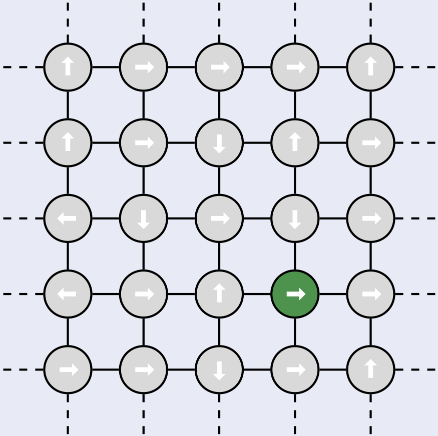
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

TU Graz
Graz University of Technology



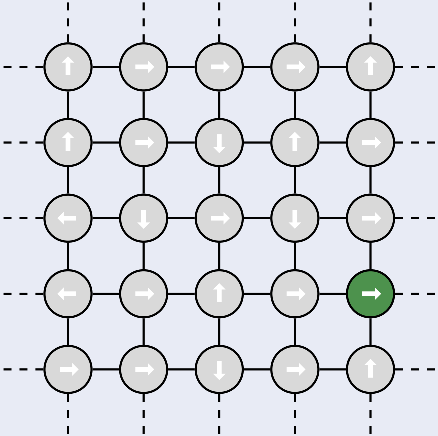
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



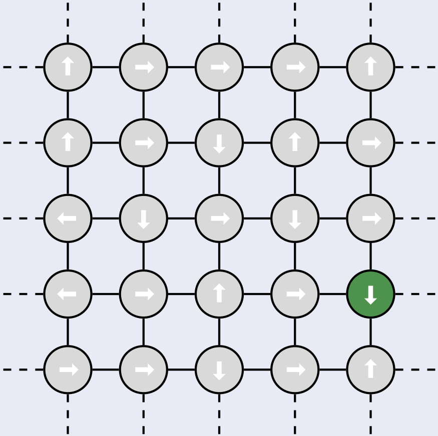
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



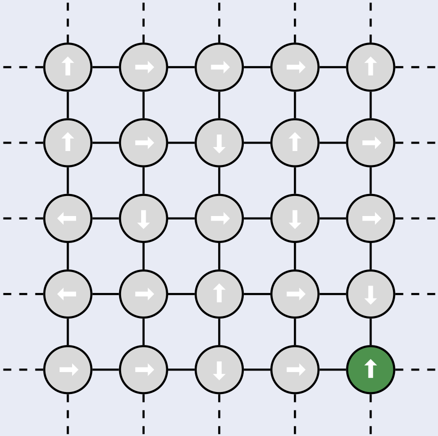
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



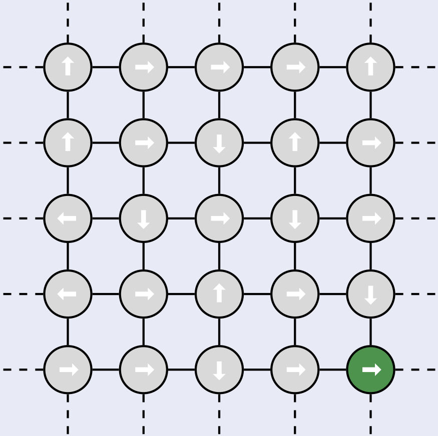
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



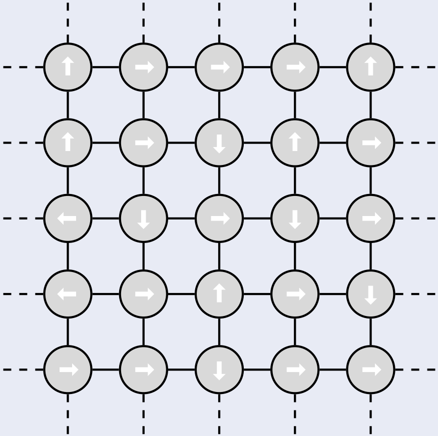
Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Example: $G = \mathbb{Z}^2$, clockwise rotor sequence



Rotor-Router Aggregation: like IDLA, but with rotor-router walks.
$R(n)$ is called rotor-router cluster with n particles. Shape of $R(n)$?

Aggregation on Comb Lattices

Graz University of Technology