Random Processes on Groups

Regularity and Phase Transitions

TU Graz - Aix-Marseille Université

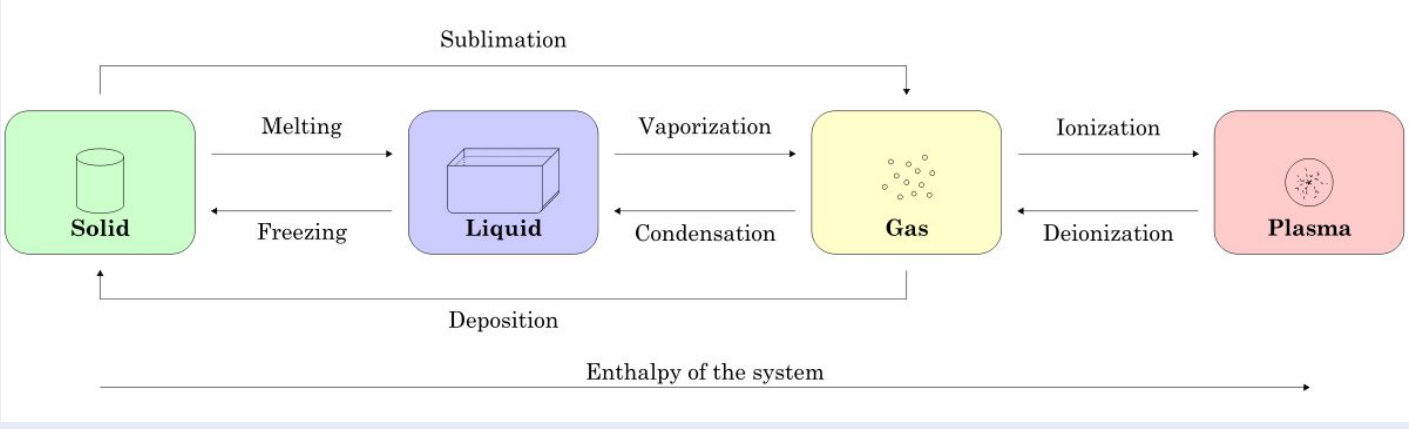
Project Nr. FR11/2014 (Graz-Marseille)

Phase transitions are everywhere

Just one example:

- Phase transition is a phenomenon observed in nature and theoretical models in many different contexts.
- It deals with a sudden change in the properties of a large structure caused by altering a critical parameter.
- Water has three phase transitions:

$$\text{ice} \rightarrow \text{water} \rightarrow \text{steam} \rightarrow \text{plasma}.$$



Phase transitions in mathematics

- Random discrete structures model real life situations; population growth, magnetization, porosity of rocks,...
- Phase transition in these models (random graphs, random graph processes, Ising/Potts model, percolation) has captured the attention of many scientists in recent years.
- These models are mathematically fascinating and have many applications.
- Deep theorems have been proved, but many problems of central importance remain unsolved and to be explored.
- Most of the existing studies connect probability theory with statistical physics.

Phase transitions in our project

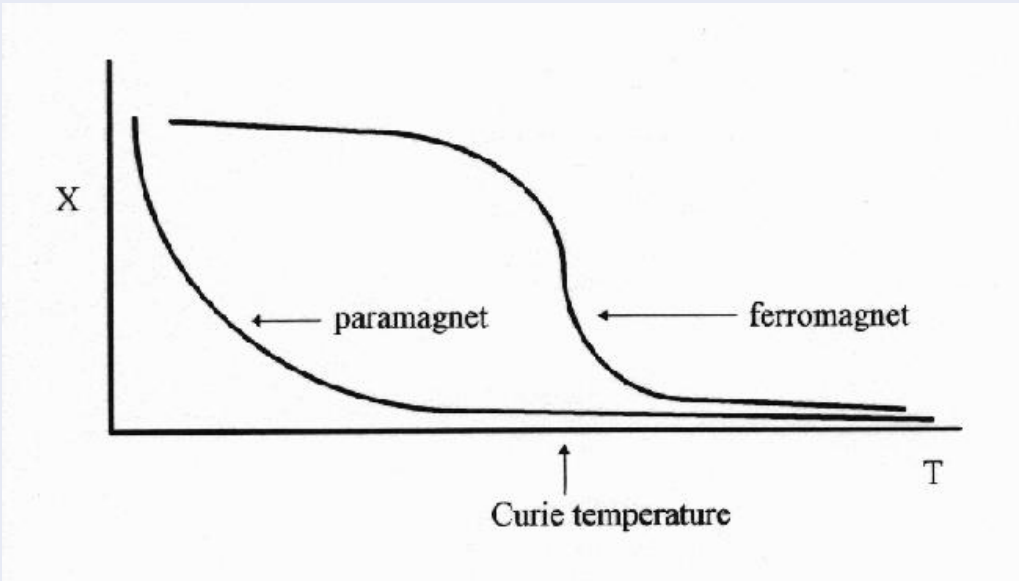
Goal of the project

- Connect probability theory, algebra, and geometry in the context of phase transitions.
- It deals with several stochastic processes such as:
  - Random walks (models various diffusion processes)
  - Branching random walks (models spatial population evolution)
  - Percolation (models porosity)
  - Ising model (models magnetism)

  For these processes, we shall analyze the concept of phase transitions and universality (or regularity).

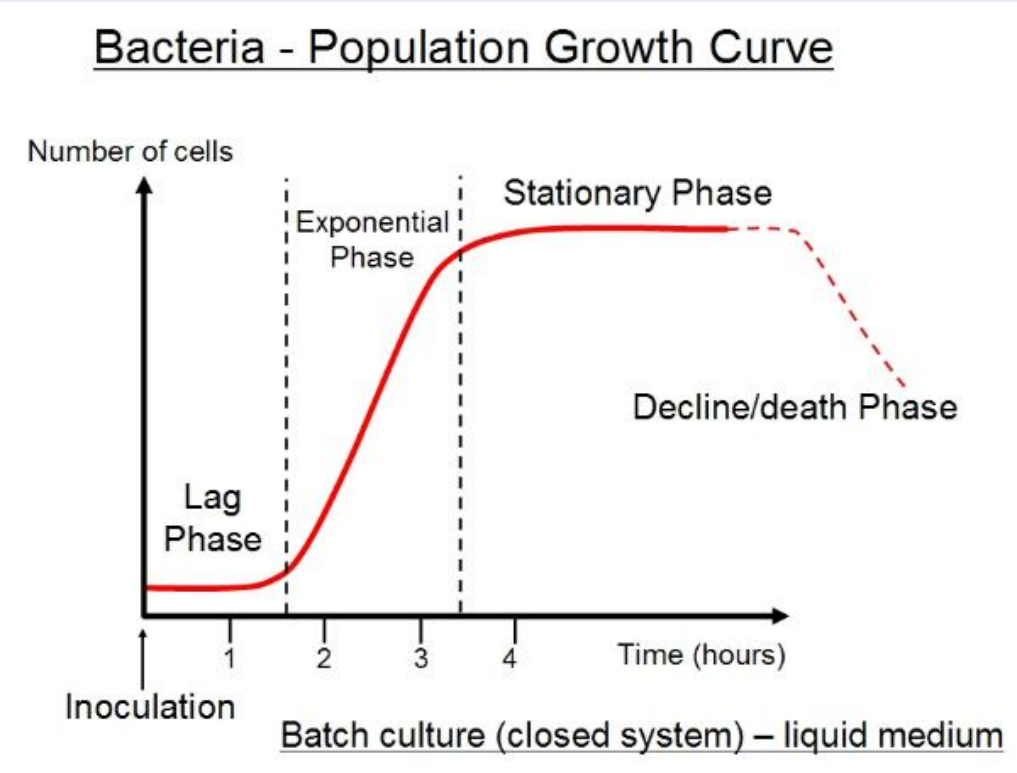
One model to analyze: Ising model

- It is one of the most studied models in physics.
- It models magnetism. More precisely: a block of iron in a magnetic field become magnetized. If the magnetic field is turned of, then
  - either the iron remains magnetized,
  - or the magnetization vanished.
- The block remains magnetized if and only if the temperature is below the Curie temperature of 700°C.



Branching random walks

- Such processes models spatial population evolution.
- A population may explode if its birth rate is above 1 and otherwise becomes extinct.
- The nature of the population therefore depends critically on the balance of creating and annihilating particles.



Thank you for your attention!