Amenability of horocyclic products

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Definitions:

a. **Amenability of a graph:**([4], see [1] for group-amenability)

The graph $G = \langle V, E \rangle$ is amenable, if there is a (*Følner-*)sequence $\{Q_n\}$ of finite subsets $Q_n \subset V$, such that the isoperimetric ratios converge to zero, i.e.

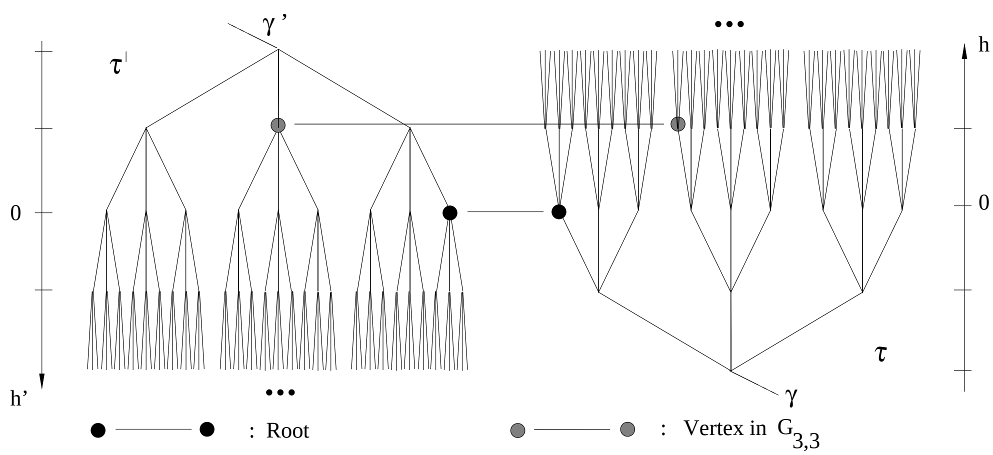
$$\frac{|\partial Q_n|}{|Q_n|} \ \rightarrow \ 0, \text{ as } n \rightarrow \infty,$$

where $\partial Q_n = \{ \ \{x, y\} \in E \mid x \in Q_n, \ y \in V \setminus Q_n \ \}$ is the *edge-boundary* of Q_n.

b. **Horocyclic Products:**([8], see [2] for interpretation as *Lamplighter-group*)

Let $\tau' = \langle V', E' \rangle$ and $\tau = \langle V, E \rangle$ be two infinite, locally finite, simple, rooted trees, pointed at infinity with Busemannfunctions h' and h, respectively. Then the horocyclic product $\tau' \circ \tau$ is defined by the graph $\bar{G} = \langle \bar{V}, \bar{E} \rangle$ with

$$\begin{aligned} \bar{V} &= \{ \ \langle v', v \rangle \ \in \ V' \times V \mid \ h'(v') \ + \ h(v) \ = \ 0 \ \}, \\ \bar{E} &= \{ \ \{\langle v', v \rangle, \ \langle w', w \rangle\} \subset \ V' \times V \mid v' \sim_{\tau'} w', \ v \sim_{\tau} w \ \}. \end{aligned}$$



Question: For which type of trees is $\tau' \circ \tau$ amenable?

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