Random walks, random configurations, and horocyclic products

Proposal for an FWF project by

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1. Introduction

After returning to Austria in fall, 1999 (from 1988 to 1999, I had been professor at the University of Milan, Italy), I started to build up a small research group in my area. In view of the fact that the (few) assistant positions at Institut für Mathematik C were already held by tenured persons working in different fields, this could an can only be accomplished via project work, by engagement of PostDocs and PhD students, in combination with efforts to bring established workers in my field to Graz for shorter periods as visiting professors, or within other exchange programs (Socrates/Erasmus, RDSES-program of ESF,...).

Several young people have been working in my projects here in Graz, see below in §2. However, only one regular assistant position became vacant in the last 6 years, so that the continuation of research activities within future projects is vital for the group.

At the center of my efforts was the previous project "Asymptotic properties of random walks on graphs", FWF P15577, of which the one proposed here is to be considered a direct continuation along more specific lines. (One of my current project collaborators - Dr. Florian Sobieczky - has also presented his own new proposal which officially, according to FWF rules, figures as a successor proposal to FWF P15577, but goes in a rather different direction than the present one.)

The original proposal text of FWF P15577 is still available online at the web address

`http://www.math.tugraz.at/~woess/fwf.ps`

so that here I shall avoid a long outline of the general state of art of random walk theory.

The basic philosophy is to study the *interplay* between stochastic properties of random walks on graphs and groups (which are typically infinite and discrete in my case) and analytic aspects

on one hand, and the geometric, algebraic, or combinatorial features of the state spaces on which these random walks evolve.

While FWF P15577 had a rather general nature and a variety of proposed sub-themes of research, my plan for the next future is to study in detail the following three themes:

(1) Lamplighter random walks (random walks on wreath products), in particular on trees and other "non-amenable" structures;
(2) internal diffusion limited aggregation on non-homogeneous graphs;
(3) horocyclic products of structures other than trees.

The first two items could be circumscribed by the appealing title "random configurations driven by random walks", while it needs some additional explanation why and how the third item is related with this title, and in particular, with "lamplighters". More details will be given below in §4.

2. Recent and current work, projects, and collaborators

More or less at the same time as FWF P15577, I managed to obtain funding for two more projects, each one for a single PostDoc: one was the FWF Schrödinger return fellowship Nr. R2Mat "Spectral Problems and Noncommutative Probability Theory", held by Dr. Franz Lehner 2001–2004, and the other one was the EU Marie Curie PostDoc fellowship HPMF-CT-2002-02137 held by Dr. Sara Brofferio 2003–2004 (Sara has now a Maître de Conference position at Univ. Paris-Orsay).

An overview over these and related activities at Insitut für Mathematik C can be found in the enclosed report entitled

"Activities at Institut für Mathematik C since September 1, 1999".

In the sequel, I'll refer to this as *MathCreport*.

On the scientific level, it is not reasonable to describe the progress of these projects completely separated from each other. Thus, the following short review is a presentation of the collaborators in those three projects along with an outline of their work, referring to the reference list at the end of *MathCreport*.

Dr. Franz Lehner obtained his Ph.D. at Univ. Paris 6 under the supervision of Gilles Pisier. He works in free probability theory, which is close to my own research interests in view of the link with the study of random walks on free products of groups (in an old paper, I anticipated Voiculescu's famous formula for the free sum of random variables in the specific context of random walks). Lehner's main achievements concern the application of free probability to the study of spectra of convolution operators, and the development of the theory of cumulants within the non-commutative framework. See references [15], [26], [27], [28], [36] and [37] in *MathCreport*. Lehner has now a regular assistant position at Institut für Mathematik C and has recently obtained his "Habilitiation".

Dr. Sara Brofferio was my master student at Univ. Milano and then went to Univ. Paris 6, where she completed her PhD (advisor: the late Martine Babillot). She then came to Graz as a PostDoc, first within the FWF Project P15577. Subsequently, she was awarded the EU Marie Curie fellowship HPMF-CT-2002-02137 at Graz. Her precious presence at Graz was a major input for a lively research environment. Her achievements during the stay at Graz concern random walks on "affine" groups on one hand, and random walks on the lamplighter group (and DL-graphs) on the other, and last but not least, on internal diffusion limited aggregation on groups with exponential growth. See references [9], [11], [18], [22], [29], [30], [39] in *MathCreport*.

Recall that the funding of FWF P15577 corresponded to one PostDoc and one PhD position. After Sara moved to the Curie fellowship, the vacant PostDoc position was first held by

Dr. Ronald Ortner. Ortner had done his PhD at Salzburg University (advisor: Johann Linhart) in Discrete Geometry and then worked in the Industry for a year before coming to Graz. He occupied a half assistant position at TU Graz plus a half PostDoc position within FWF P15577 in the academic year 2002/03. Thereafter he accepted an offer for a 6 year assistant at Montanuniversität Leoben. He concluded work on cogrowth and non-backtracking random walks, see [16] in *MathCreport*.

During the search of a successor, I decided to admit a small shift in the topics towards the inclusion of more Harmonic Analysis, the reason being that there was an excellent candidate, namely

Dr. Markus Neuhauser. He had done his PhD at Munich University under the supervision of Bachir Bekka and Günter Schlichting on Kazdhan's property T. Thereafter he had held a PostDoc Position at Univ. Neuchatel (with Alain Valette) before coming to Graz in fall, 2003. He now (since October, 2005) holds an assistant position at Universität Göttingen. Besides continuing his work on property T, he collaborated with A. Karlsson (Stockholm) on a conjecture of Bass (geometric group theory) and on heat kernels and zeta identities on cyclic groups, all topics with an obvious link with, resp. importance for the study of random walks. See references [12], [14], [19], [23] in *MathCreport*.

In addition, Neuhauser and I are still continuing work on a long and substantial paper with the title "Horocyclic porducts of trees", see reference [5] in *MathCreport*. This is joint work with **Dr. Laurent Bartholdi** (Lausanne), who is one of the external collaborators of FWF P15577. In a previous paper ([20] in *MathCreport*), Bartholdi and I had obtained a detailed understanding of the spectrum of random walk operators on lamplighter groups and Diestel-Leader graphs. The latter are horocyclic products of two trees, and the three of us are now studying horocyclic products of an arbitrary number of trees. This involves a number of different aspects: the description of the full automorphism groups of those graphs, the question when one of these graphs is a Cayley graph of some fintely generated group and how this group is related to "lamplighters", properties of those groups like homotopy, presentation and growth function, spectrum of random walk operators and asymptotics of transition probabilities, and the Poisson boundary of random walk.

This work of mine with Bartholdi and Neuhauser, together with the paper [18] (in *MathCreport*) with Brofferio, are for me the most satisfactory outcomes of FWF P15577 ([18] is also shared by Sara's Curie fellowship), since there the strong interplay between structure theory and random walks plays a major role. A continuation of this line of research is also one of the main ingredients of the present proposal.

Dr. Elmar Teufl has been a close collaborator ever since I came to Graz, first as a "Studienassistent" (tutor) and then on a 50% assistant employment, while working on his PhD thesis under the supervision of P. M. Grabner. His main research interests concern the applicaton of methods from complex function theory (iteration theory, singularity analysis) to random walk questions on fractal graphs, see [25] in *MathCreport*. After finishing PhD and Civil Service, Dr. Teufl has held a 50% PostDoc position within FWF P15577 from April through September, 2005. He has gone to Bielefeld University with a EU Curie fellowship in December, 2005, where – also due to some "diplomacy" by myself – he is collaborating with Prof. Alexander Grigoryan (who has just moved from Imperial College to Bielefeld; indeed, Teufl's fellowship first was for Imperial College). Teufl is about to conclude two papers that are attributed to FWF P15577, [3] and [4] in *MathCreport*, regarding random walks and additional features of trees, resp. graphs, with self-similarity properties.

Regarding the proposed PhD position within FWF P15577, this had been intended for **Mr. Wilfried Huss**, who at the time of the proposal for FWF P15577 was about to start to work on his master thesis ("Diplomarbeit"). However, due to health problems, this work was delayed.

In the meantime, Mr. Huss has finished an excellent master thesis on "Internal Diffusion Limited Aggregation", containing in particular new experimental results regarding this process on trees with finitely many cone types, which is the subject of a publication in preparation, [1] in *MathCreport*, funded by a 2months' research grant of TU Graz after Huss has been awarded his Dipl.Ing. (≡ master) degree with honours.

When the abovementioned delay became clear, the remaining project period appeared insufficient for employing a different PhD student. Also, at about the same time, I managed to obtain funding from TU Graz for another PhD position (involving teaching), now held by Mr. Gilch (see below). Hence I decided, within the freedom granted by FWF in this resepct, to look for other PostDocs to collaborate in FWF P15577. Among several candidates who responded to internet and email postings, there was one who presented himself with an excellent talk:

Dr. Florian Sobieczky had studied Physics at Göttingen, and after a short period in industry, he did a PhD in Mathematics under the supervision of Prof. Hering (Göttingen) while holding a temporary assistant position at TU Berlin. He is a project collaborator in FWF P15577 since October, 2004 (1.9.–31.12.2005 "on leave" in Bochum). He introduced a new topic to my research group, namely the study of random walk on percolation clusters. While originally he had done this on the classical grids, following my suggestion he included arbitrary (unimodular) transitive graphs in his considerations. He has managed to conclude a first paper on this subject, [8] in *MathCreport*.

This last line of research is going to be reinforced from January to July, 2005, by **Dr. Adam Timar**, who is about to finish his PhD under the supervision of Russell Lyons at Indiana University (Bloomington). Timar has obtained very interesting results on percolation on non-unimodular graphs and related questions, see e.g. his recent papers [T1, T2, T3]. It is intersting to note that the main result of my paper with Soardi [S-W] from 1990 has become an important basic ingredient in the study of percolation on transitive graphs, see Benjamini, Lyons, Peres and Schramm [B-L-P-S] as a basic reference. As a matter of fact, the same result also stands at the origin of a good part of my present work on random walks that has been performed within FWF P15577 as well as previously, in particular the papers [5], [11], [13], [17], [18], [20], [21], [22], [30], [38] in *MathCreport*.

A short word on the *external collaborators* of FWF P15577, listed on page 2 of *MathCreport*: all of these persons have visited Graz at least once for a longer stay during the last years. Each of them, and in particular **V. A. Kaimanovich** and **L. Saloff-Coste**, have been permantent sources of stimulation and suggestions. A particular success has been the collaboration within FWF P15577 of **L. Bartholdi**, both in the work [5] and [20] in *MathCreport* and in the stimulating ideas resulting from discussions on almost all other research questions that we have dealt with.

Completing the picture of young collaborators in my research group, **Mr. Lorenz Gilch** is currently holding a 4year PhD assistant position, including teaching duties. His research concerns methods for computing the rate of escape of random walks on free products of groups and of lamplighter random walks on homogeneous trees; see [6] and [7] in *MathCreport*.

At this point, I would also like to briefly mention some of my own work of the recent years that is not all directly part of one of the three projects, but is of course thematically closely related with them. The long (more than 50 pages) paper with **L. Saloff-Coste**, [17] in *MathCreport*, deals with the (delicate) extension of the results of [S-W] to the non-discrete setting where a locally compact group acts co-compactly on a proper metric space: spectral radius and norm of transition operators are related with amenability and unimodularity of the acting group. A sequel to this paper is in preparation, [2] in *MathCreport,* where we apply these results to diffusion on trees on one hand, and to convolution operators on groups on the other.

Two visits of **D.I. Cartwright** to Graz an one of myself to Sydney resulted in two completely different papers: the very recent [10] regards the averaging operator on the 1-skeleton of a

network, whose spectrum is computed in terms of the transition operator on the 0-skeleton. The other one, [24] in *MathCreport*, concerns isotropic random walks on $\tilde{A}_d$-buildings, which are studied via isotropic harmonic analysis in the spirit of the classical work of Sawyer [S] on homogeneous trees. This work was recently completed in the excellent dissertation of a PhD student of D.I. Cartwright, **Dr. James Parkinson**, who considers a much larger class of buildings, see his papers [P1, P2, P3].

As a matter of fact, Dr. Parkinson is the principal candidate for a PostDoc position within the presently proposed project.

Regarding the scientific atmosphere at the insitute, please also see §10 of *MathCreport*, containing a detailed list of all the guests during the last years who stayed for one 1–2 weeks up to (for visiting professors) one month.

At present (academic year 2005/06), the research group consists of **Dr. Lehner, Mr. Gilch** and myself as employees of TU Graz, **Dr. Sobieczky** and **Dr. Timar** (starting with 15.1.2006) as FWF PostDocs, and **Prof. Maura Salvatori** (on leave from Universitá di Milano) as a visiting Professor from 1.10.2005 until 31.5.2006. **Prof. V.A. Kaimanovich** (Bremen) is scheduled to come for a month in January/February 2006, and and a one-week visit of **Prof. Nina Gantert** (Münster) is planned for June 2006. Visiting professors in the academic year 2006/07 (hopefully again funded by TU Graz) will probably be **Prof. Tullio Ceccherini-Silberstein** (Benevento/Roma) and **Prof. Alexander Bendikov** (Wroclaw).

3. State of the art

As mentioned in §1, I will not repeat the general outline regarding random walks on infinite graphs and groups, my approach to it, and the general state of the art up to 2001, as given in the proposal of FWF P15577.

Here, I give a short overview of some recent, important developments and selected references. Note that this short overview is restricted to work on random walks on infinite graphs and groups, where these structures are deterministic objects, and the transition matrix is determined in terms of the structures. That is, here I do not touch the various other lines of research such as random walk on random graphs, reinforced or self-avoiding random walk, and so on, nor do I give a detailed review of ongoing work on random walk in purely Euclidean setting.

The last year saw an ongoing, powerful research regarding the methods from Functional and Stochastic Analysis that link heat kernel and transition probability asymptotics with isoperimetric inequalities, elliptic and parabolic Harnack inequalities, and with escape time exponents. This is closely related with analogous questions regarding Riemannian manifolds. See the work of Barlow, Coulhon and Grigor'yan [B-C-G], Grigor'yan and Telcs [G-T1, G-T2], Coulhon, Grigor'yan and Pittet [C-G-P] for general results of this type. An innovative variant of the general Gaussian estimate for random walks (Markov chains) that are not necessarily symmetric (or reversible) is due to Mathieu [M].

It is noteworthy that the above line of research has received new input from persons who have been working on Brownian motion and analysis on fractals, who also provided new classes of expamples that exhibit interesitng behaviour. See e.g. Barlow [B] and Hambly and Kumagai [H-K1, H-K2]. This also shows how sharp and new tools have been developed for the study of random walks, and how much the topic has expanded since the early 1980ies when this type of research was basically restricted to random walks on groups only. As a matter of fact, also Kaimanovich [K], one of the major experts in random walks on groups, has recently exhibited a very interesting approach to the study of fractal-like graphs via an elegant construction of hyperbolic graphs whose boundary is a fractal (the Sierpinski triangle in [K]).

Also within the research regarding random walks on groups there is a noteworthy progress. One of the fascinating subjects is the study of links between the algebraic structure and the

asymptotic type of transition probabilities. A major step, regarding solvable groups, is the paper by Pittet and Saloff-Coste [P-SC3], see also their preceding note [P-SC2]: on solvable groups with finite Prüfer rank, the return probabilities behave like $\exp(-n^{1/3})$. Previously, in [P-SC1], they had shown that on other clasees of solvable groups, namely wreath products, there is a large variety of different possible asymptotic types of random walks.

In the last few years, one could see emerge a very strong young reseracher in the area: Erschler [E1, E2, E3, E4] has obtained a wealth of striking new results, in particular regarding the interplay between entropy of random walks, rate of escape, and growth of groups. Among other, via random walks, in [E3] she exhibits examples of groups with subexponential growth that grow faster than $\exp(n^\alpha)$ for any $\alpha < 1$. In [E1], continuing preceding work, she gives estimates of the rate of escape on iterated wreath products.

The rate of escape and a local limit theorem for random walks on wreath products (lamplighter groups) have also been studied by a former Ph. D. student of Saloff-Coste, David Revelle, see e.g. [R2, R3].

In several respects, the study of random walks and transition operators on graphs and of Brownian motion and the Laplacian on manifolds evolves along similar lines. This is present in the work of several of the authors mentioned above. Another input from the side of differential geometry came in recent years from Kotani and Sundada [K-S1, K-S2] and also Shirai [K-S-S], who have been interested in *crystal lattices*, that is, graphs on which an abelian group acts with finitely many orbits. One of their interests was the "intrinsic geometry" associated with random walks on crystal lattices: they investigated the geometric meaning of the leading constant in the local limit theorem as well as the pseudometric appearing in the Gaussian off-diagonal estimate, and the spectrum and the density of states of the transition operator.

In the 1990ies and the first years of the new millenium, a major new impetus in random walk theory came from a strong group of younger Israeli-US mathematicians: Benjamini, Lyons, Pemantle, Peres, Schramm and collaborators; see the proposal text of FWF P15577. Their main interests appear now a little bit shifted towards various other areas within modern probability, among which percolation, uniform spanning trees, conformal invariance, and Schramm-Loewner evolution. The latter two themes are part of the breathtaking series of work by Lawler, Schramm and Werner, see e.g. [L-S-W] for one of the papers of that series. Two other recent highlights of that group of persons are the papers of Lyons, Peres and Schramm [L-P-S] and Benjamini, Kesten, Peres and Schramm [B-K-P-S]. The forthcoming book by Lyons with Peres [L-P] contains – among various topics – several very readable chapters on different aspects of random walk theory; for me, it has already served as an excellent basis for several courses.

Let me also mention briefly one aspect of the study of spectral theory of random walk transition operators on groups and graphs. The lamplighter group $\mathbb{Z}_q \wr \mathbb{Z}^2$ was the first example where an associated "Simple random walk" operator on an infinite group has a pure point spectrum: this was obtained by Grigorchuk and Żuk [G-Z] (for $q = 2$) and (for arbitrary q) by Dicks and Schick [D-S]. (Via Diestel-Leader graphs, this has been put in a more general and at the same time more elementary frame by Bartholdi and myself in [20], *MathCreport.*)

Finally, let me say a few words (with a few selected references) about *random walks in Austria.* I was introduced to this beautiful subject by P. Gerl (Salzburg) who unfortunately has ceased his research activities in that field with [G2]. In Linz, Ch. Takacs has produced a few interesting papers on random walk in random environment, the last one [Ta] dating back to 2002. The two strong combinatorics groups of Ch. Krattenthaler at Vienna University and M. Drmota at Vienna University of Technology repeatedly touch the topic in the context of counting lattice or other walks with specified properties, e.g in [K-G-V] and [Dr]. The famous Hungarian probabilist P. Révész was professor at Vienna University of Technology. He has given many contributions to classical lattice random walks from a pure probabilistic viewpoint,

see e.g. [Re]. From the side of Ergodic Theory, K. Schmidt (Vienna Univ.) and his collaborator G. Greschonig [G-S] have investigated recurrence of random walks with stationary increments. And at TU Graz, my colleague P. Grabner has studied Brownian motion on fractals [G-T], and subsequent to our collaboration on random walk on the Sierpinski graph [G-W], he has carried out and supervised further work on random walks on fractal graphs, where powerful methods from complex function theory play a major role, see e.g. Teufl [Te] and [25] in *MathCreport.*

4. Project research: details

A. Lamplighter random walks.

This and the topic **C** below could be subsumded under the nice title **random configurations driven by random walks**, and also topic **B** below has a (less direct) link with this subject. Think of a graph (locally finite, connected), where at each vertex there is a lamp that may be in q different states of intensity, including $0 \equiv$ "off". Initially, all lamps are "off". A "lamplighter" starts a random walk along X, and at each step, he can make a step to a neighbour vertex in X, or he may modify at random the state of the lamp at his actual position (or, more generally, he may perform a combination of both types of action). The state space of this Markov chain is not just X, but it is the set $\mathbb{Z}_q \wr X$ of all pairs (x, η), where $x \in X$ is the current *position*, and $\eta : X \to \mathbb{Z}_q = \{0, \dots, q-1\}$ which is the current *configuration* of states of the lamps. The support $\mathsf{supp}(\eta)$ has to be finite, since only finitely many lamps are "on" at any time. (Instead of $\mathbb{Z}_q$, one may take any other rooted graph, whose root represents the "off" state of the lamps, see [13] in *MathCreport.*)

Thus, at time n (after n steps), we observe the pair of random elements (X_n, Y_n), where X_n is the position and Y_n the random configuration of lamps. This is a Markov chain as well as its first projection (X_n), while (Y_n) alone is not Markovian.

In algebraic terms, lamplighter random walks are random walks on wreath products of groups. Currently, a lot of research is being done on this subject, see e.g. [P-SC1], [E1], [E3], [R2], [R3], [G-Z], [D-S]. The research on this topic goes back to Kaimanovich and Vershik [K-V], who were the first to observe some of the specifically interesting features of this class of random walks (relating to bounded harmonic functions, rate of escape, and asymptotics of return probabilities). See also Lyons, Pemantle and Peres [L-P-P] and the papers from 2001 by Erschler [E1] and Revelle [R1].

The basic case is the one where $X = \mathbb{Z}$ (as a graph, this is the two-way infinite path). Results regarding random walks on $\mathbb{Z}_q \wr \mathbb{Z}$ consitute one of the major successes in the preceding project FWF P15577, together with Brofferio's Curie fellowship. These results became possible via a detailed understanding of the geometry of a Cayley graph of $\mathbb{Z}_q \wr \mathbb{Z}$. This is the *Diestel-Leader graph* $\mathsf{DL}(q, q)$. Diestel-Leader [D-L] have introduced the graph $\mathsf{DL}(2, 3)$ as a candidate example for answering a problem posed by Woess in [S-W]: is there a vertex-transitive graph that is not quasi-isometric with any Cayley graph ? In general, if $q, r \geq 2$, then $\mathsf{DL}(q, r)$ is constructed as follows. Take two homogeneous trees $\mathbb{T}_q$ and T_r of degree q and r, respectively. In each one, select a boundary point and the associated *Busemann (horocycle)* function $\mathfrak{h} : \mathbb{T} \to \mathbb{Z}$. Its level sets are the *horocycles*. Then $\mathsf{DL}(q, r)$ is the *horocyclic product*

$$\mathsf{DL}(q, r) = \{x_1 x_2 \in \mathbb{T}_q \times \mathbb{T}_r : \mathfrak{h}(x_1) + \mathfrak{h}(x_2) = 0\},$$

a subgraph of the direct product graph $\mathbb{T}_q \times \mathbb{T}_r$. This is a one-ended, vertex-transitive graph (careful: it is by no means quasi-isometric with a tree !). Its full automorphism group is amenable, and – as a locally compact group – it is unimodular if and only if $r = q$.

In a series of papers [21], [20], [18], [11] *(MathCreport)*, we were able to exploit the structural features of $\mathsf{DL}(q, r)$ for obtaining detailed results about classes of random walks: minimal harmonic functions [19], [9], the spectrum of transition operators and precise asymptotics of return

probabilities [18], and the complete Martin compactification of nearest neighbour random walks [16]. In previous work, Bertacchi [Be1] (a former PhD student of mine at Milano) had already determined the rate of escape (which coincides with the one of the projected random walk on $\mathbb{Z}$) and - more delicate - central limit theorem for a large class of walks on Diestel-Leader graphs.

Within the proposed project, I plan to pursue the study of lamplighter random walks on **trees**, in particular homogenous trees and trees with finitely many cone types in the sense Nagnibeda and Woess, see [34] in *(MathCreport)* and the description on p4 of the proposal text of FWF P15577. The interesting fact that the spectrum of simple random walk on $\mathbb{Z}_q \wr \mathbb{Z}$, resp. $\mathsf{DL}(q,q)$, is pure point appears to rely on the fact that the "base graph" $\mathbb{Z}$ where the lamplighter moves is a tree. It appears to ba a reasonable working hypothesis, to be verified rigorously, that the same is true for lampligher walks on (at least, homogenous) trees.

As a matter of fact, lamplighter walks have so far been studied almost exclusively on amenable groups (graphs). Homogeneous trees are the simplest non-amenable graphs where this study can be undertaken. Some first results have been obtained by Gilch, [6] in *(MathCreport)*, who gives rather precise estimates of the rate of escape (while explicit computation appears extremely hard), and in (unpublished) conversations of myself with Karlsson, identifying the Poisson boundary (bounded harmonic functions). A specifically interesting question on general nonamenable groups, in analogy with the many results in the amenable setting mentioned in §3, is to study the asymptotic type of $p^{(n)}(x,x)/\rho(P)^n$, where the transition matrix P is symmetric and $\rho(P) < 1$ is the spectral radius of the random walk. In many known specific cases (free groups, free products, etc.) the last sequence has asymptotic type $n^{-3/2}$. For nonamenable wreath products, resp. other semidirect products, new tools have to be developed, and lamplighter walks on homogeneous trees are the best initial example for attacking this problem.

Beyond group-invariant walks, in the same spirit a class of basic structures that appear to lend themselves to such a study of lamplighter walks are the trees with finitely many cone types. Among them, a paricularly interesting class of trees are the **comb lattices**. They are spanning trees of the integer lattices $\mathbb{Z}^d$, but simple random walk on them exhibits interesting behaviour in several respects. The main reason is the non-validity of the so-called Einstein relation for typical exponents appearing in random walk asymptotics. See Gerl [G1], the work of my former PhD students Bertacchi and Zucca [B-Z], [Be2], and a recent paper by Krishnapur and Peres [K-P] for some of the peculiar features of comb lattices.

B. Horocyclic products.

In the work in progress of Bartholdi, Neuhauser and myself, [5] in *MathCreport*, we study the construction analogous to that of $\mathsf{DL}(q,r)$, where instead of 2 one takes d homogenous trees $\mathbb{T}_{q_1}, \dots, \mathbb{T}_{q_d}$ and their horocyclic product

$$\mathsf{DL}(q_1, \dots, q_d) = \{x_1 \cdots x_d \in \mathbb{T}_{q_1} \times \dots \times \mathbb{T}_{q_d} : \mathfrak{h}(x_1) + \dots + \mathfrak{h}(x_d) = 0\},$$

equipped with a natural qraph structure. In other words, this is a horosphere in a direct product of trees. Several questions become much harder here, and in our (already long paper) we can settle part of them: the algebraic and geometric structure, spectrum of simple random walk (it is always pure point !), asymptotics of return probabilities, and Poisson boundary (i.e., the bounded harmonic functions). Let me remark that also in this case, a (restricted) "lamplighter" interpretation is available.

Other questions, such as the minimal positive harmonic functions, asymptotics of the Green function, Martin compactification are intriguing and considerably harder than for $\mathsf{DL}(q,r)$. Also, the graph metric of those higher dimensional horcyclic products is quite complicated, so that rate of escape and central limit theorem become more delicate and interesting topics to be studied in future work within the proposed project. Within the context of horocyclic products of trees, I also plan to untertake a study of random walks on a specific class of totally disconnected, locally

compact groups, the groups $\mathsf{Sol}_d(\mathbb{Q}_p)$, where $\mathbb{Q}_p$ are the p-adic numbers (they might be replaced by any other non-archimdean local field). In case $d = 2$, this is the semidirect product of the multiplicative group $\mathbb{Q}_p^*$ with the additive group $\mathbb{Q}_p^2$, where the action of the first on the latter is given by $\big(\lambda, (\alpha_1, \alpha_2)\big) \mapsto (\lambda\alpha_1, \lambda^{-1}\alpha_2)$, compare with the final question posed in Pittet and Saloff-Coste [P-SC3]. This group acts on $\mathsf{DL}(p, p)$, and for larger d, there is a similar group acting on the horocyclic product of d copies of $\mathbb{T}_p$.

More so, the above construction of horocyclic products is not restricted to trees. On $\widetilde{A}_d$ buildings, the analog of the "Busemann function" with respect to a boundary point is a mapping $\mathfrak{h}$ from the building to $\mathbb{Z}^d$, and one can construct the horocyclic product of two or more buildings of the same dimension in the same way as above. Here, again, one encounters very interesting problems, where it is clear that a good understanding of the algebraic and geometric structure must go hand in hand with the study of random walks. Based on my work with Cartwright – [24] in *MathCreport* – and his own papers [P1, P2, P3], **James Parkinson** is an ideal person for attacking this range of questions.

This study requires a considerable initial part of "pure" structure theory: what is the full automorphism group, when do we get a Cayley graph of a finitely generated group, and what is this group. Also, the (geo)metric structure needs specific consideration, and one has to clarify how and which aspects of harmonic analysis on trees and buildings has to be involved here.

A preliminary step in this approach should be the study of random walks on a single $\widetilde{A}_d$ building that are not necessarily isotropic in the sense of [24] in *MathCreport*, but *semi-isotropic*, i.e., invariant under the stabilizer of a boundary point of the building. In the case $d = 1$, when the building is a tree, a detailed study of such random walks was carried out by Cartwright, Kaimanovich and Woess [C-K-W] and subsequently refined by Brofferio in [22], *MathCreport*. Basic questions when $d \geq 2$ concern e.g. rate of escape and related limit theorems, convergence to the boundary, Dirichlet problem at infinity for harmonic functions, Poisson boundary.

C. Internal diffusion limited aggregation.

This process (IDLA) was already mentioned in the proposal for FWF P15577 and was also studied within the Curie fellowship of Sara Brofferio. Consider a locally finite graph with a root ("source") o that emits particles. Each particle performs a random walk, independently from the previous ones, until it first finds an unoccupied vertex (site), which it then occupies. Then the next particle starts its random walk, an so on. At the n-th step, n particles will have occupied a connected subgraph A_n of size n, the aggregation cluster at time n. The main question here is to find a link between the asymptotic shape of A_n as $n \to \infty$ and the geometry of the underlying graph.

The model goes back to Diaconis and Fulton [D-F] (on $\mathbb{Z}$) and was studied on $\mathbb{Z}^d$ in detail by Lawler, Bramson and Griffeath [L-B-G] and Lawler [L]. In his PhD thesis (I was one of the "referees") and resulting papers, Blachère [Bl1], [Bl2] refined the latter results and also undertook first steps to extend the study to groups other than $\mathbb{Z}^d$. Within Brofferio's Curie project here in Graz, Blachère and Brofferio, [9] in *MathCreport*, have started a collaboration and managed to obtain a very complete understanding of IDLA on groups with exponential growth. At the same time, **Wilfried Huss** has completed his Master thesis, where besides a study of the exisiting work on IDLA, he undertook a clever experimental work on IDLA on trees with finitely many cone types. I now propose to pursue the rigourous study of IDLA on these trees as a basic class of graphs where no group acts transitively or even with finitely many orbits. As a matter of fact, this had already been included in the proposal for FWF P15577: recall the delay caused by health problems of Mr. Huss, which are now overcome. The experimental results (contents of a little publication, in progress) will serve as an excellent basis for the study of IDLA in particular on the comb lattices (the natural spanning trees of

$\mathbb{Z}^d$) mentioned above. Indeed, on the latter, we already have a rather precise idea about the geometry of the clusters A_n, which are "diamond-shaped" rather than spherical as in the case of the lattices $\mathbb{Z}^d$. This research appears feasible, interesting and new, ideally suited for starting a PhD thesis.

D. Further interesting problems.

Let me outline a few further interesting problems that appear to be within the range of the competences of myself and my (present as well as future) collaborators.

D.1. Diffusion on $\mathsf{HT}(q,r)$. Horocyclic products can also be built in another way. A particulary interesting example is that of the horocyclic product of hyperbolic upper half plane with the homogeneous tree $\mathbb{T}_r$. This is a two-dimensional complex obtained by glueing horizontal "strips" of the upper half plane together in a tree-like manner. The amenable Baumslag-Solitar group $\langle a, b \mid ab = b^p\rangle$ has a natural action on $\mathsf{HT}(q,q)$ which is discrete and co-compact. For a picture, see e.g. Farb and Mosher [F-M], who use this space to study quasi-isometries of the Baumslag-Solitar groups. On this space, one can define a natural two-dimensional Laplacian wihich projects onto a Laplacian (possibly with drift) on the hyperbolic plane, and to a second order differential operator on the one-skeleton of the tree. A first interesting problem here is to determine all positive harmonic functions for the Laplacian. The geometric behaviour of random walks on the Baumslag-Solitar groups can also be best understood via its action on that space. As a (difficult) long term aim is to use $\mathsf{HT}(q,q)$ and its geometric boundary at infinity for determining the Martin boundary of the latter random walks.

D.2. Dirichlet finite harmonic functions on "lamplighter trees". For lamplighter random walks on homogeneous trees, it apppears to be an interesting problem to see whether there are non-constant Dirichlet finite harmonic functions, that is, harmonic functions whose differences along the edges of the corresponding graph (Cayley graph of the lamplighter group over the tree) are square summable. Are they all constant ? This interesting question (which might even have a simple answer) was recently formulated by **Anders Karlsson (Stockholm)** and myself.

The research topics outlined here may be slightly more than a group of two or three young researchers collaborating with me may be able to solve completely within three years. Let me repeat here a phrase that I formulated in the proposal for FWF 15577: "I believe we all know that the utmost details of one's mathematical research are not easily predictable in every detail in advance." Thus, once more, the above is a reservoir of interesting questions, and I believe that my background is sufficient to elaborate solutions of a good part of them in collaboration with two or three young mathematicians.

5. Project duration

This project should start in October 2006 and have a duration of three years.

6. Project personnel

I am applying (again, as for FWFP15577) for one PostDoc and one PhD position.

The candidate for the PostDoc position is **Dr. James Parkinson.** He was already mentione above in §2. Parkinson has been a PhD student of **Donald I. Cartwright** at Sydney University. My research cooperation with Donald Cartwright dates back to 1989; since then we have published 5 joint papers (the last one is still in press). Among those, there is the

relatively recent paper on "Isotropic random walks in a building of type $\tilde{A}_d$". It was the basis for an important part of Parkinsons thesis, namely his paper [P3], where he elaborates a far reaching study of random walks on the vast class of affine buildings. This and his other two papers [P1, P2], accepted for puglication in very good journals, show that he has an excellent understanding both of the structure and of the harmonic analysis on buildings that is necessary to obtain those probabilisitc results. Thus, he appears to be the ideal person to undertake, in particular, the research work outlined in §4.**B** on horocyclic products, as well as the related problems concerning "semi-isotropic" random walks on buildings that are invariant under the group of isometries which fix a boundary point.

The second position that I'm applying for is a PhD position. The main candidate here is **Dipl.-Ing. Wilfried Huss.** As mentioned above, he has already gained very good experimental insight in IDLA on trees, in particular comb lattices, in his master (diploma) thesis. Right now, he is elaborating a report on this experimental study within a short (2 months) fellowship awarded by TU Graz. In 2006, he will make his civil service and is scheduled to return to TU Graz for PhD in October.

The educational aspect of the present project, which is manifested in this application for a PhD position, is closely linked with current developments at my University: TU Graz is currently starting a cooperation in the area of Natural Sciences, including Mathemtatics, with the University of Graz. Part of this collaboration is going to be a common Doctoral School in Mathematics. In this framework, it is likely that TU Graz will call for "co-financed" PhD projects. Thus, it appears to be reasonable to split the PhD position for which I am applying here in two halves, such that the requested FWF funding can cover two PhD positions, where the second half of each is funded by TU Graz on the basis of the Natural Science cooperation project. In this context, the research topic for a second PhD student will be the one of §4.**A**, "lamplighter random walks on trees". I do not yet propose a definite candidate for this part of the project, since it appears better to advertise such a position and to look for the best one among the applicants.

7. International collaboration

As in the precending project, my good international contacts will play an important role in the continued development of the research group. As a matter of fact, the random walk comunity is rather an international network consisting of small research groups in many different places, than consisting of barycenters with lot of persons working in similar directions in single countries (such as, for example, number theory in Austria, or PDE in Italy, etc.). During the last decade, this random walk comunity has established a good exchange (in part, I hope, also due to my efforts in the organization of conferences ad workshops in 1997, 2001, 2004, 2006 and 2006) both among its members and with other areas such as Harmonic Analysis, Potential Theory or – in particular – Geometric Group Theory, where people have become interested in random walks in the context of the search for geometric invariants.

Thus, other experts with whom it necessary to have a continued exchange do not come from nearby cities within Austria, but from abroad. There are several possible financial sources for getting these persons to Graz (or vice versa, to visit them): visiting professorships at TU Graz (where the payment has decreased, however) ore, vice versa, abroad (where, however, the administration of TU Graz has recently raised difficulties regarding the salary during such stays), very few short visits in Graz funded by the corresponding (small) budget of TU Graz, exchange of teaching personnel within the EU Erasmus/Socrates programme, the visitor exchange within the ESF programme "RDSES" (until 2007), etc.

Within the present project, I also propose to establish specific international collaborations on the basis of visits of a few weeks in either direction (guests coming to Graz, or project

collaborators from Graz going abroad). This collaboration is of course specifically intended for the research proposed in this project. In particular, the following persons from abroad should be part of this exchange within the work on the topics proposed in this project.

First of all, there are two younger persons:

(1) **Sara Brofferio**, now at Paris-Orsay, is very bright and deep, the mathematical interaction with her has always been a pleasure, and her knowledge in all three of the proposed topics is profound: during her fellowship in Graz, we have collaborated on lamplighter random walks, resp. horocyclic products of two trees. Previously, she has worked on random walks on affine groups, and, in particular, on the affine group of a tree. The latter is the stabilizer of and end of the tree, while one of the subjects proposed here is to search for detailed understanding of random walks on the stabilizer group of a boundary point of a building. Finally, she has also produced (with Blachère) a very nice paper on IDLA. In this case, the collaboration should primarily consist in two visits of Sara at our insitute, for a total of about 3 weeks during the project. It appears crucial to maintain permanent contacts with Sara Brofferio, who in a sense has not ceased to be a member-at-distance of my research group.

(2) **Laurent Bartholdi**, previously at Berkeley and now at EPF Lausanne, has already had a noteworthy impact on previous work within FWF P15577. This is not only manifested in the two papers [5] and [20] *MathCreport,* but he also has been a permanent partner for exchange of ideas on various questions. Besides visits in the framework of FWF P15577, he has also come to Graz in October 2004 as a visiting professor, giving an excellent course on groups and automata. His competences regard the more algebraic sides of the present project, also including spectral theory and other analytic aspects. Since part of the proposed project work consists in extensions of our paper [5] on horocyclic products, we can expect very fruitful contributions from Bartholdi. As in the case of Sara Brofferio, the collaboration should primarily consist in two visits, for a total of about 3 weeks, at our insitute.

(3) **Donald Cartwright** from the University of Sydney certainly plays a major role in the background of this project, both as the doctoral "father" of J. Parkinson and as one of the few worldwide experts in harmonic analysis on buildings. His expertise, along with his patient and thorough style of carrying out joint research, are very important for the success of the proposed work. In the past, Cartwright came to Graz as a visiting professor rather than within the financial framework of the previous projects. However, since TU Graz now gives lower remuneration for visiting professorships, it will be necessary to combine the latter with partial co-funding from projects. It will be highly desirable to bring Cartwright to Graz once more as a visiting professor for 4–6 weeks in the "middle" phase of the project, of which 2 funded within this project.

(4) **Laurent Saloff-Coste,** professor at Cornell University, is a leading figure in current research on random walks on groups (and not only). He has gained a wide international reputation in the last decade and is or has been member of the editorial boards of several top journals (Annals of Probability, Prob. Theory and Related Fields, Stochastic Processes and Appl., Mathematische Zeitschrift). Together with Ch. Pittet and others he has been active in exploring the link between group structure and asymptotic type of random walks. We have written 3 joint papers, a 4th one is in preparation. Saloff-Coste has come to TU Graz as a visiting professor in 2003. Within the present project, his expertise is particularly precious for topic A (lamplighter random walks). I plan to bring him to Graz for a few weeks during some of his visits to Europe and, if his time permits, as a visiting professor in a later phase of the project. On the other hand, it also appears desirable from the viewpoint of the scientific formation of the younger project collaborators to send them to Cornell for shorter periods of approximately two weeks.

(5) Finally, **Vadim A. Kaimanovich** (previously at Rennes) is now professor at the International University Bremen. He is *the* international expert on measure theoretic boundary

theory. Besides our joint research throughout the last 15 years (4 joint papers), our collaboration has also comprised the organization of conferences (e.g., the Random Walk Semester at the Schrödinger Institute in Vienna in 2001, and the forthcoming workshop on Discrete Probability at the same instution). Kaimanovich has always provided input on various questions around boundary theory, amenability, rate of escape, etc. I plan to bring him to Graz within this project for a total of 2 weeks, while additional visits my be arranged on other bases.

8. Research plan, dissemination

In the first year, the postdoc research assistant (i.e., J. Parkinson) should mainly focus on the study of random walks on the stabilizer group of a boundary point of a building, since this is not only interesting in itself, but also a basic requirement for studying horocyclic products of buildings. Subsequently, his efforts should concentrate on the latter structures.

The PhD student (i.e., W. Huss) should start by searching for rigorous results concerning IDLA on the "comb lattices". Subsequently, the same process is to be studied on trees with finitely many cone types which are irreducible (i.e., whose graph of cone types is strongly connected).

In the (likely) case there is a second PhD position within the Natural Sciences project at Graz, that PhD student should focus on lamplighter random walks on trees, beginning with extensions of the current work of Mr. Gilch, and then turning to questions regarding harmonic functions on one hand and non-homogenous trees on the other.

In all three cases, it is realistic that the project progress allows a successful "screening" of intermediate success after one year, which then in turn should lead to a detailed update for the research plan in the subsequent period.

Dissemination of new mathematical results following current habits is quick and efficient: preprints are posted not only on the Insitute's webpage, resp. the individual webpages of team members, but also in the Mathematics ArXiv, where they gain immediate international visibility well before they appear in a regular mathematical journal. Besides this, conference talks, exchange with visitors, etc., are usually also quite efficient.

Outside the scientific comunity, dissemination is hard and its fruitfulness might even be doubted sometimes. Public consideration of Mathematics by journals, radio, TV, or similar is scarse and usually only refers to topics of applied mathematics with immediate non-mathematical profit. One useful platform in Austria was the "Science week", which unfortunately has been called off and substituted by new attemps from "above" whose effects are not yet clear.

9. Importance and impact

As mentioned elsewhere, "Random Walks" as a research subject is popular, but rather spread out in many smaller research groups all over the world. Among those small groups there is the one at Insitute für Mathematik C, lead by myself. I believe to have gained a good reputation within Austrian mathematics as well as abroad. The present project is vital for the group's survival beyond a one-man-performance.

This also clarifies the impact of this project within the scientific comunity: to maintain high quality and visibility of the (basically) only random walk research group in Austria.

10. Requested funding

The requested funding is primarily for a Post Doc fellowship for three years (**J. Parkinson**) and for a Ph.D. fellowship for the same duration. (**W. Huss**).

Furthermore, I plan to bring the following persons to Graz, using project funds for periods of 10-14 days, each (while additional stays should be financed by other sources):

L. Sara Brofferio in autumn/winter 2006/07 and approximately one year later; collaboration regarding IDLA related with topic **C** in §4,

L. Bartholdi in spring/summer 2007 and 2009; collaboration regarding horocyclic products related with §4 **A, B**,

D. I. Cartwright in summer 2007; collaboration regarding random walks on buildings and their horocyclic products related with §4 **B**,

V. A. Kaimanovich in summer 2008; collaboration regarding boundary theory related with §4 **A, B**,

L. Saloff-Coste autumn/winter 2008-09 ; collaboration regarding lamplighter random walks, IDLA and diffusion on $\mathsf{HT}(q, r)$ related with §4 **A, C, D.1**.

For each of these five persons, I am estimating average travel costs only within Europe at an amount of 400 EURO. The costs for their stays are estimated on the basis of the official "Reisegebührenvorschrift" (guidelines for travel refunding), that is, 52 EURO/(day+night) each for Cartwright, Kaimanovich and Saloff-Coste; 43,20 EURO/(day+night) each for Bartholdi and Brofferio.

Furthermore, both team members should get the opportunity to visit Cornell University within the indicated project collaboration for two weeks (or more, by use of different sources). Here, the support towards travel costs should amount to 800 Euro each, while the funding of the stays according to the "Reisegebührenvorschrift" amount to 72,90 EURO/(day+night). These visits should take place in the final phase of this project, when it will be desirable that team members go abroad and improve their own contacts for finding possibilities for a continuation of their activities after the project.

Further exchange in either direction can be funded by visiting professorships, Erasmus exchange (for the PhD student as well as for teaching staff), or other sources such as (for example) currently the one-week exchange visits within the RDSES program of ESF.

	1st year	2nd year	3rd year
PostDoc salary (Parkinson)	51.570,–	51.570,–	51.570,–
PhD salary (Huss)	30.150,–	30.150,–	30.150,–
Brofferio to Graz	832,–	832,–	
Bartholdi to Graz	832,–		832,–
Cartwright to Graz		1.128,–	
Kaimanovich to Graz		1.128,–	
Saloff-Coste to Graz			1.128,–
PostDoc to Cornell			1.820,5
PhD to Cornell			1.820,5
total (Euro)	83.384,–	84.808,–	87.321,–

Grand total: 255.513,– Euro

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[T1] Timar, A.: *Percolation on Nonunimodular Graphs*, preprint, Indiana Univ. (2005).

[T2] Timar, A.: *Ends in Free Minimal Spanning Forests*, Annals of Probability, to appear.

[T3] Timar, A.: *Cutsets in Infinite Graphs*, Combinatorics, Probability and Computing, to appear.

[W] Woess, W.: *Random Walks on Infinite Graphs and Groups*, Cambridge Tracts in Mathematics **138**, Cambridge University Press, Cambridge, 2000.

12. Curriculum vitae and recent publications of Wolfgang Woess

Born in Vienna (Austria) 23 July 1954.

Studies of Mathematics at the Technical University of Vienna and the Universities of Munich and Salzburg.

1978: Diploma (≡ Master) degree in Mathematics at the Technical University of Vienna.

1980: Ph. D. in Matematics at the University of Salzburg (supervisor: P. Gerl).

1980–81: Research grant at the University of Salzburg.

1982–88: University assistant (Assistant professor) at the Institute of Mathematics and Applied Geometry of the Montanuniversität Leoben (Austria).

1984–85: on leave, research fellowship (sponsored by the Italian CNR) at the Department of Mathematics of the University of Rome (research group of A. Figà-Talamanca).

1985: "Habilitation" in Mathematics at the University of Salzburg.

April 1986: Visiting professor at the Universities of Rome and Milan.

1987: Biennial prize of the Austrian Mathematical Society.

Spring 1987: Visits at several universities in the U.S. (Washington University - St. Louis, City University of New York) and Canada (Université de Montréal, Simon Fraser University).

1988–1994: Associate professor for Mathematical Analysis at the University of Milan (Italy).

Summer 1989: Visiting professor at the University of Sydney (Australia).

1989 and 1990: Referee for NSF research grant applications.

1990–1999: Associate Editor of the journal "Circuits, Systems and Signal Processing" (Birkhäuser).

May/June 1991: Visiting professor at the University of Salzburg (Austria).

April–May 1992: Visits at several universities in the U.S. (Harvard Univ., CUNY, Washington Univ. - St. Louis, USC and UCLA).

May 1993: Visiting professor at Université Paris 7.

1994–1999: Full professor of Probability at the University of Milan (Italy) and (1998–1999) at the second University of Milan.

April/May 1995: Visiting professor at the University of Salzburg (Austria).

May 1996: Visiting professor at Université de Rennes-I.

April/May 1997: Visiting professor at the University of Vienna.

June 1997: Organization (together with V. A. Kaimanovich, M. A. Picardello and L. Saloff-Coste) of the INdAM conference "Random Walks and Discrete Potential Theory", Cortona (Italy).

May 1998: Visiting professor at the Technical University of Graz.

Fall semester 1998/99: on leave from Milano, visiting professor at the Universities of Vienna and Linz.

Since September 1999: Full professor of Mathematics at the Technical University of Graz.

August/September 2000: Visiting professor at Cornell University.

January 2000 - December 2003: secretary of the Austrian Mathematical Society (ÖMG)

Organization of the special semester "2001 - Random Walks" at the Schrödinger Insitute in Vienna (together with V. A. Kaimanovich and K. Schmidt) and of the conference "2001 - Fractals in Graz" (June 2001, together with P. Grabner).

April/June 2002: Visiting professor at Insitut H. Poincaré, Centre Emile Borel.

Member of the program committee of the congress of the Austrian Mathematical Society in collaboration with UMI and SIMAI, (Bozen/Bolzano, September 2003).

April 2004: Visiting professor at the University of Sydney

Organization of the conference "Geometric Group Theory, Random Walks, and Harmonic Analysis" (June 2004, Cortona, Italy).

2005 - : Head of the committee for doctoral studies at TU Graz.

Organization of the special session "Stochastic Analysis on Metric Spaces" (together with L. Saloff-Coste and K.-Th. Sturm) at the 2nd joint meeting of AMS, DMV, ÖMG (June 2005, Mainz, Germany)

Organization of the "RSDES/ESI Educational Workshop on Discrete Probability" (together with V. A. Kaimanovich and K. Schmidt) at the Erwin Schrödinger Insitute (March 2006, Vienna, Austria).

Recent publications

Following FWF suggestions, the 7 most important publications are indicated with boldface & underlined numbers.

[1] W. Woess: *Random Walks on Infinite Graphs and Groups,* Cambridge Tracts in Mathematics **138**, Cambridge University Press, 334+xi pages, 2000.

[2] S. Brofferio and W. Woess: *On transience of card shuffling,* Proc. Amer. Math. Soc. 129 (2001) 1513–1519.

[3] W. Woess: *Heat diffusion on homogeneous trees,* Bollettino Un. Mat. It. 4-B (2001) 703–709 and erratum (caused by printer), Bollettino Un. Mat. It. 5-B (2002) 259–260.

[4] V. A. Kaimanovich and W. Woess: *Boundary and entropy of space homogeneous Markov chains,* Ann. Probab. 30 (2002) 323–363.

[5] T. Nagnibeda and W. Woess: *Random walks on trees with finitely many cone types,* J. Theoret. Probab. 15 (2002) 399–438.

[6] T. Ceccherini-Silberstein and W. Woess: *Growth and ergodicity of context-free languages,* Transactions Amer. Math. Soc. 354 (2002) 4597–4625.

[7] T. Ceccherini-Silberstein and W. Woess: *Growth-sensitivity of context-free languages,* Theoretical Comp. Sci. 307 (2003), 103–116.

[8] W. Woess: *Generating function techniques for random walks on graphs,* in "Heat Kernels and Analysis on Manifolds, Graphs, and Metric Spaces", P. Auscher, Th. Coulhon and A. Grigor'yan, eds. Contemporary Math. 338 (2003) 391–423.

[9] P. M. A. Grabner, W. Woess (editors): *Fractals in Graz 2001: Analysis - Dynamics - Geometry - Stochastics,* Proceedings (Graz 2001), Birkhäuser, Basel, 283+iii pages, 2003.

[10] V. A. Kaimanovich with K. Schmidt and W. Woess (editors): *Random Walks and Geometry,* Proceedings (ESI, Vienna, 2001), de Gruyter, Berlin, 532+x pages, 2004.

[11] D. I. Cartwright and W. Woess: *Isotropic random walks in a building of type $\tilde{A}_d$,* Math. Zeitschrift 247 (2004) 101–135.

[12] W. Woess: *Lamplighters, Diestel-Leader graphs, random walks, and harmonic functions,* Combinatorics, Probability & Computing 14 (2005) 415–433.

[13] L. Bartholdi and W. Woess: *Spectral computations on lamplighter groups and Diestel-Leader graphs,* J. Fourier Analysis Appl. 11 (2005) 175–202.

[14] S. Brofferio and W. Woess: *Green kernel estimates and the full Martin boundary for random walks on lamplighter groups and Diestel-Leader graphs,* Annales Inst. H. Poincaré (Prob. & Stat.) **41** (2005) 1101–1123.

[15] L. Saloff-Coste and W. Woess: *Transition operators on co-compact G-spaces,* Revista Matematica Iberoamericana, to appear.

[16] R. Ortner and W. Woess: *Non-backtracking random walks and cogrowth of graphs,* Canadian J. Math., to appear.

[17] W. Woess: *A note on the norms of transition operators on lamplighter graphs and groups,* Int. J. Algebra and Computation, in print.

[18] S. Brofferio and W. Woess: *Positive harmonic functions for semi-isotropic random walks on trees, lamplighter groups, and DL-graphs,* Potential Analysis, in print.

[19] D. I. Cartwright and W. Woess: *The spectrum of the averaging operator on a network (metric graph),* Illinois J. Math., to appear.

[20] L. Bartholdi, M. Neuhauser and W. Woess: *Horocyclic products of trees,* preprint, TU Graz.

[21] L. Saloff-Coste and W. Woess: *Computations of spectral radii on cocompact G-spaces,* in preparation, TU Graz.

13. Curriculum vitae and publications of James Parkinson

Personal Details

- Name: James William Parkinson
- Date of birth: 16/10/79
- Address: 13/458 Edgecliff Rd
  Edgecliff 2027
  NSW Australia
- Email: jamesp@maths.usyd.edu.au
- Web: http://www.maths.usyd.edu.au/u/jamesp/

Education

1. 2002–2005, Ph.D. (Mathematics), University of Sydney.
   Thesis title: *Buildings and Hecke Algebras.*
   Supervisor: Dr. Donald Cartwright.
2. 1998–2001, Bachelor of Science (Adv) with First Class Honours in Pure Mathematics, University of Sydney.
   Thesis title: *Regularised Products and Zeta Functions.*

Employment History

1. **Lecturing:**
   - 2006, Jan–Feb, I will be the lecturer in charge for a first year undergraduate calculus course in summer school. Duties include 6 hours of lecturing each week, preparation of tutorials and exam, and marking.
   - 2005, Semester 2, lecturer for a first year undergraduate course in algebra. Duties include two lectures per week, tutorials, consultation, preparation of assignments, quizzes, computer project and exam, marking, and webmaster for the course webpage.
   - 2005, Jan–Feb, lecturer in charge for a second year undergraduate course 'Matrix Applications' over summer school. Duties include four hours of lecturing per week, preparation of tutorials, assignments, quizzes and exam, and marking of quizzes, assignments and the exam.
   - 2004, Jan–Feb, lecturer in charge for a second year undergraduate course 'Matrix Applications', with duties as above.
   - 2003, substitute lecturer for five lectures of a second year advanced course on vector calculus and complex variables.
2. **Tutoring:** 2001–2005, employed as a casual tutor (supervising exercise and practice classes and marking assignments) for classes including undergraduate courses in algebra, calculus and discrete mathematics.

Conferences and Seminars

1. 2005:
   - Buildings Seminar (at Sydney University). Each week I gave a one hour talk to a group of five academics who attend the seminar. Topics include buildings and Hecke algebras, and most lectures are adapted from material in my thesis.
   - Australian Mathematical Society annual conference. I gave a talk on *Spherical Harmonic Analysis on Affine Buildings*.
   - Algebra Seminar (Sydney University). I gave a one hour talk *Buildings and Hecke Algebras* on my thesis.
   - Sydney University 'Sums' talk. I gave a one hour talk on *Random walks on trees and grids*.
2. 2004:
   - Australian Mathematical Society annual conference. I gave a talk called *Averaging Operators on Buildings*.

Publications

1. J. Parkinson. *Buildings and Hecke Algebras*, Journal of Algebra, to appear (approximately 55 pages).
2. J. Parkinson. *Spherical Harmonic Analysis on Affine Buildings*, Math. Zeitschrift, to appear (approximately 45 pages).

Preprints

1. J. Parkinson. *Isotropic Random Walks on Affine Buildings*, submitted to Annales de l'Institut Fourier (approximately 35 pages).
2. J. Parkinson. *Hecke Structure Constants*, in preparation.

14. Curriculum vitae and publications of Wilfried Huss

Name: Dipl.-Ing. Wilfried Huss
Personal data:

- **Date of birth:** 12.06.1979
- **Place of birth:** Klagenfurt
- **Nationality:** Austria
- **Religion:** Roman Catholic
- **Marital status:** Single
- **Parents:** Josef Huss, Gertrude Huss (born Wouk)
- **E-Mail:** huss@finanz.math.tugraz.at

Education:

- Elementary school Sittersdorf
- 1997 Graduation at the BG and BRG Vlkermarkt
- October 1997 start of the studies of technical mathematics (information processing) at the Technical University of Graz.
- November 21, 2005 diploma exam.
- December 2005 - January 2006 research scholarship of the Federal Ministry for Education, Science and Culture.
- January 2006 - September 2006 civil ervice at the "Lebenshilfe Graz und Umgebung - Voitsberg".

Languages: German, English, Slovenian
Teaching Experience:

- October 1999 - June 2001 Teaching Assistant at the Department of Mathematics A.
- October 2001 - June 2002 Halfemployed Teaching Assistant at the Department of Mathematics C.
- October 2002 - February 2004 Teaching Assistant at the Department of Mathematics C.

Qualifications:

- Programming skills in the languagues: C, C++, Qt, Perl, Python, PHP, SML, Haskell, Java
- Knowledge of UNIX administration and programming tools (Apache, CVS, Subversion, Autoconf, Automake).
- TeX/Latex
- Computer Algebra Systems: Mathematica, Maple
- Since October 2004 maintainer of KViewShell/KDVI, a multiformat document viewer for the KDE desktop environment.

Title of Diploma Thesis: Internal diffusion-limited aggregation
Publication: Internal diffusion-limited aggregation on comb lattices: simulation and experimental analysis (in preparation)