

Final report - stand alone project FWF P31889-N35

Walks and boundaries – a wide range

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1. INTRODUCTION

In view of my retirement (October 1, 2022), this has been the last of my four FWF stand alone projects in the period 2002-2023. They have served as the crucial “backbone” for building and maintaining a research group in my areas of research at Graz University of Technology. Each of those projects was equipped with the funding for one PostDoc and one PhD student, but the group had up to 12 members.

Let me explain my way of operating. One of my aims has been to support to young researchers in their difficult “precarious” years within the relatively wide range of topics of my work. With the project funding, I could provide a haven, with careful eyes on different funding sources, occasionally shifting collaborators from one type of funding to another one. The years of the pandemic made this more complicated and required a high degree of flexibility and several alterations in the employment statuses of the collaborators.

Thus I could enlarge my group and offer research opportunities to more than twice as many persons as provided directly by the FWF funding. In particular, the support from the present project goes beyond the direct funding, where some of the collaborators were directly funded only for several months or only in part, then continuing within the group on the basis of other funding that I managed to find; something that would not have been possible in absence of the FWF funding. Thus, the large number of publications which acknowledge at least partial support is also due to the possibilities provided project.

2. THE PROJECT’S OVERALL SCIENTIFIC CONCEPT AND GOALS

The five topics of the original project proposal were all concerned with “walks”, namely, random walks and their potential theory, as well as combinatorial aspects of walks with motivation coming from statistical physics. I quote from the project proposal: “Let me clarify that the above [here: below] is a reservoir of interesting questions.” I have re-ordered them in relation with the significance of the outcomes.

A. The study of the nature of the formal language associated with self-avoiding walks on vertex-transitive graphs, in particular finitely generated groups – to link self-avoiding walks (originally coming from theoretical chemistry and related with statistical physics) with formal language theory is a new approach which I proposed some years ago.

B. Branching random walk studies the evolution of a population which moves according to a random walk, while increasing according to a Galton-Watson process. The goal has been to study the random sequence of the empirical distributions of the population and their boundary behaviour on graphs and groups, in particular groups with “hyperbolic” properties.

C. A topic related with Martin boundary concerns the general integral representation of polyharmonic functions with arbitrary complex eigenvalues in the resolvent set of the underlying

random walk transition operator on trees as well as on general finite graphs, and subsequently also in the continuous setting of the Poincaré disk.

Two further topics of the proposal have in the end received less consideration:

D. “Sliced hyperbolic plane” is a strip complex where the hyperbolic upper half plane is subdivided into isometric horizontal strips. The associated Laplace operators can be seen in terms of random walks on matrix groups which are non-discrete and disconnected. The aim to describe the associated Martin boundary turned out to be very hard and is the subject of still ongoing attempts.

E. Ultrametric spaces arise as boundaries of trees. The plan was to extend the study of the Markov processes induced by hierarchical Laplacians to random perturbations. It was not realised because the envisaged project collaborator (Wojciech Cygan) could not re-join the research group.

Instead, further nice and fertile research work was brought into the project by young team members; see below.

3. FWF DOCTORAL PROGRAM (DK) “DISCRETE MATHEMATICS”

This is mentioned for getting a better view on the environment in which the present project was embedded. This “Doktoratskolleg” (equivalent of the German “Graduiertenkolleg”) was established in 2010. It is a joint project of altogether 15 mathematicians (“faculty members”) from TU Graz and the Universities of Graz (KFU) and Leoben (MUL), in 3 subsequent funding periods with substantial funding from the FWF and from TU + KFU. I was the “speaker” (director) of this programme until Sept. 30, 2022 (retirement). It is currently in its phase-out period until June 30, 2024.

The PhD students were internationally recruited. Similarly to the first and second phase, in the last phase the salaries plus generous travel allowances for 12 of them were funded by FWF, 3 positions were directly funded by TU and two by KFU. In addition there were/are further associated PhD students with salaries from different sources, but with the same travel allowances, etc, for a (varying) total of up to 25.

For more details, see

<http://www.math.tugraz.at/discrete>

The project employees have interacted strongly with this doctoral program, with mutual benefits.

4. PROGRAM DURATION, PERSONNEL AND RESULTS

This section combines the chronological evolution of the project with a presentation of the project collaborators and the most significant project publications.

Remark. For the mentioned publications, I always refer to the chronological numbering, as to be found on the webpage

<https://www.math.tugraz.at/~woess/FWF-P31889-publ.html>

The project started officially on April 1, 2019. I could take a sabbatical semester in 2019 (March – September), in which I undertook 3 research visits for approximately one month each.

A first part of the project research already started in March 2019, namely my collaboration with **Marc Peigné** in Tours, France, leading to the paper number **6**,

- M. Peigné - W. Woess: “Recurrence of 2-dimensional queueing processes, and random walk exit times from the quadrant”, *Annals of Applied Probability* 31 (2021) 2519–2537.

Contrary to my preceding projects (were I tried to reserve almost the entire funding for the young collaborators), this time I used part of the available travel funds for two stays abroad. In May 2019, I visited **Vadim A. Kaimanovich** at University of Ottawa. We pursued the research on branching random walks, leading to paper number **20**,

- V. A. Kaimanovich - W. Woess: “Limit distributions of branching Markov chains”, *Annales Inst. H. Poincaré Prob. & Stat.* 59 (2023) 1951–1983.

The second stay, in June 2019, was in Reykjavik, where I started a collaboration with **Rögnvaldur Möller** on amalgamation of G -spaces via group actions on augmented trees. This is still not finalised; the state of my contributions to this work is documented on my personal publications website <https://www.math.tugraz.at/~woess/publications-ww> (number 105).

From October 1, 2019 – September 30, 2020, the first project post doc was **Stjepan Šebek**. He had graduated from Zagreb University and has been collaborating intensively with Woiciech Cygan (Wrocław/Dresden). Cygan had been envisaged as a project member, but in the end could not join us and recommended Šebek. The type of research where Šebek could display his strengths and high productivity was somewhat different from the originally planned topic, concerning the range of stable random walks and subordinated random walks. Based on my attitude to provide a free environment for research to young collaborators, his employment was very fruitful. As a representative for his 5 contributions where full or partial project support is acknowledged, there is paper number **7**

- W. Cygan - St. Šebek: “Transition probability estimates for subordinate random walks”, *Mathematische Nachrichten* 294 (2021) 518–558.

In the period March 1, 2020 – April 4, 2021, my PhD student **Christian Lindorfer** was employed half-time within the project, while I had funding from TU Graz for the other half. He had already started on April 1, 2018 as an associated PhD in the doctoral programme “Discrete Mathematics” (see above).

Note that (1) project PhD students get a contract for 30 hours per week, with the idea that they can also payment. (2) His part-time employment is typical for my way of operating: with the project at hand, I was able to guarantee Lindorfer an employment as a PhD student for the necessary number of years. Successfully looking for additional funding, I could then keep him as a precious collaborator while at the same time being able to employ further project personnel. Let me stress here again that in this perspective, it is clear that Lindorfer as well as others contributed substantially to the project, no matter from which funding parts of their salaries came in parts of the period. In the period May 1 – September 20, 2021 he was again fully funded by

TU Graz as an associated student within the DK, and he graduated with distinction in September, 2021. From October 1 – December 31, 2021, Lindorfer was fully employed as a project post doc. From January 1 – December 31, 2022, he was only employed part time (25%), and served as the coordinator of the DK “Discrete Mathematics” (75%). Finally, in the last year of the project, I still managed to employ him at 87,5% in the period January 1 – September 30, 2022, including a small upgrade of his salary at the very end. This required a careful planning of the financial resources; see also below (Oppelmayer and Hammer). Lindorfer proved to be a very talented young mathematician. He had received the student prize or the Austrian Mathematical Society for his master thesis which then was published in a Springer series, he received the ”Best paper award” 2020 of the Doctoral School Mathematics of TU+Uni Graz, and he was nominated for the Heidelberg Laureate Forum (online participation in view of the pandemic).

His publications are key contributions to the present project. I’m proud of my idea to link self-avoiding walks with formal language theory. The excellent implementation is in large part due to Lindorfer in papers **2** and **19**, along with Lehner:

- Ch. - W. Woess: “The language of self-avoiding walks”, *Combinatorica* 40 (2020) 691-720,
- F. Lehner - Ch. Lindorfer: “Self-avoiding walks and multiple context-free languages”, *Combinatorial Theory* 3, issue 1 (2023), article no. 18, 50 pp.

In the period March 1, 2020 – December 31, 2021, **Florian Thomas (Lehner)** was employed at varying part-time levels within the project. After marriage, he assumed his wife’s family name Thomas, but due to academic practice, his publications are still signed “Florian Lehner”. He had been a long-time member of my research group since his master thesis. He had held an FWF Schrödinger fellowship 2017–2019 at University of Warwick, thereafter coupled with a return grant, which then was coupled with the employment in the present project. He had obtained a tenured lecturer position at the University of Auckland, to start in summer 2020, but due to the lockdown measures he could move there only in autumn 2022, while being officially employed already since January 1, 2022. From November 1, 2020 – December 31, 2021, he was also part-time employed as the coordinator of the DK “Discrete Mathematics”. (Compare with my statement in §1 of the difficulties and required flexibility in relation with the pandemic!)

Florian Thomas (Lehner) has worked on various aspects of structural graph theory; see his webpage

<https://www.florian-lehner.net>

He has a very fruitful collaboration with Christian Lindorfer. The abovementioned paper **19** is a highlight of the project results. Another substantial paper on self-avoiding walks is in preparation:

- F. Lehner - Ch. Lindorfer - Ch. Panagiotis: “SAW is ballistic on graphs with more than one end” (forthcoming, Jan.-Feb. 2024).

(Panagiotis was a PhD student of Agelos Georgakopoulos in Warwick and subsequently PostDoc with Hugo Duminil-Copin in Geneva; he is now in Bath.)

From November 15, 2020 – April 14, 2021, **Hanna Oppelmayer** was a post doc in the project. I had known her at the beginning of 2020, when she was a PhD student of Michael Björklund

(Göteborg), and saw that her work was pertinent to the project topics. After her graduation (autumn 2020) she had planned to spend a post doc year in Israel (with Yair Hartman), but this was blocked by the pandemic. My financial plans for the project allowed for a 5 months' employment, but I did manage to find follow-up funding from TU Graz for April 15 – September 30, 2021. This can be considered as a TU Graz support for the project. Thereafter she could finally realise her stay in Israel. (Note again the need of flexibility imposed by the pandemic.) Her work is on the interplay between random walks on groups, boundaries, entropy, and group algebras. She acknowledges partial project support in two of her papers. One of them is number **10**,

- H. Oppelmayer: “Boundary entropy spectra as finite subsums”, *Stochastics and Dynamics* 21 (2021), no. 6, Paper No. 2150038, 13 pp.

Finally, from March 1 – September 30, 2023, **Stefan Hammer** was employed at 87,5% as a post doc in the project, also including a small salary upgrade at the end. Hammer had held a TU Graz assistant position at the level of a doctorand for 3 years, and was an associated PD student of the DK “Discrete Mathematics”. I was his official advisor, with major input coming from Prof. **Daniele D’Angeli**, who had spent almost 6 years in my group before moving to Rome (Univ. Cusano). Hammer’s work was on graph-theoretical indices of classes of infinite graphs related with automata groups - here, the language (and automata)-theoretic topic comes once more into play. Regarding the project outcomes, there is a paper in preparation,

- D. D’Angeli - St. Hammer: “Wiener and Szeged index of tree graph automata” (forthcoming Feb. 2024).

This concludes the presentation of the project collaborators who obtained salaries from the project funding. There were several further contributors who profited from the project environment, which led to joint publications. In those cases, the FWF financial support was only small (e.g. related to travel by myself or similar) but this does not diminish their relevance within the project. For example, there are 3 papers related to topic **C** mentioned in §2. Besides number **11**, I want to point out number **4**:

- E. Sava-Huss - W. Woess: “Boundary behaviour of λ -polyharmonic functions on regular trees”, *Annali Mat. Pura Appl.* 200 (2021) 35-50.

and

- M. Picardello - M. Salvatori - W. Woess: “Polyharmonic potential theory on the Poincaré disk”, preprint (2023).

Another paper related to the “boundary”-theme is paper number **12**:

- W. Woess: “Ratio limits and Martin boundary”, *Documenta Math.* 26 (2021) 1501-1528.

5. CAREER DEVELOPMENT OF THE FUNDED PROJECT COLLABORATORS

Stjepan Šebek has a permanent position as assistant professor, University of Zagreb, Faculty of Electrical Engineering and Computing, Croatia.

Florian Thomas (Lehner) has a permanent lecturer position at the Department of Mathematics, University of Auckland, New Zealand.

Hanna Oppelmayer has a (non-tenured) assistant prof. position at the Institute of Mathematics, University of Innsbruck, Austria, in the group of Prof. Ecaterina Sava-Huss, former PhD student and collaborator of mine.

Stefan Hammer and **Christian Lindorfer**, to my great regret, did not want to undergo the tedious precarious career phase which typically follows after PhD, spending some 1–2 years each in different places worldwide before being able to settle down after maybe 7–10 years. It is highly urgent that the academic world starts to refrain from this exaggerated demand of mobility. Excellent research can be done without this requirement, and international contacts and experiences can be gained also on the basis of visits from a fixed base point. Some young researchers like this mobility, but others don't, which by no means implies that they are less strong or promising. (Note that I state this in spite of having spent 1 + 11 years abroad myself.)

Hammer is now developing machine learning models as a software engineer at a company near Graz. **Lindorfer** is a scientific collaborator at a company in Linz, developing algorithms for biometric tools.

6. PUBLICATIONS IN PREPARATION/SUBMITTED/ACCEPTED/IN PRESS

Most have already been mentioned in the main section 4.

- M. Picardello - M. Salvatori - W. Woess: “Polyharmonic potential theory on the Poincaré disk”, submitted to Journal of Functional Analysis (positive referee report, revision required).
- D. D’Angeli - St. Hammer: “Wiener and Szeged index of tree graph automata” (forthcoming, Feb. 2024).
- F. Lehner - Ch. Lindorfer - Ch. Panagiotis: “SAW is ballistic on graphs with more than one end” (forthcoming, Jan.-Feb. 2024).
- F. Lehner - Ch. Lindorfer - R. Möller - W. Woess: “Graphs of group actions and group actions on trees” (in preparation). [This should be the final outcome/variant of my collaboration with R. Möller at the very beginning of the project.]
- F. Lehner - Ch. Lindorfer - O. Aichholzer: “Folding polyominoes into cubes” (in preparation). [The topic is not pertinent to the project, but the research was also undertaken while Lehner and Lindorfer were part of the project.]

Apart from this, during her employment in the project, Hanna Oppelmayer has started a collaboration with my former student and collaborator Prof. **Sara Brofferio** (Paris - Est), who also appeared in the initial project proposal as one of the contact partners.

7. CONFERENCE PARTICIPATIONS AND TALKS AT EXTERNAL INSTITUTIONS

In view of the FWF regulations on the length of this report, for the corresponding list I refer to the webpage

<https://www.math.tugraz.at/~woess/FWF-P31889-publ.html>