Walks and boundaries
– a wide range –

Proposal for an FWF project by

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Besides Wolfgang Woess, also Wojciech Cygan has contributed to the preparation of the proposal text.

1. Introduction

The three FWF projects "Asymptotic properties of random walks on graphs" (P15577, years 2002–2006), "Random walks, random configurations, and horocyclic products" (P19115, years 2006–2011) and "Hyperbolic structures in stochastics, graph theory and topology" (P24028, years 2012–2017) have served as the crucial "backbone" for building and maintaining a research group in my areas of research at Graz University of Technology. Each of those projects was equipped with the funding for one PostDoc and one PhD student, but the group had up to 12 members, slightly varying throughout the years.

For more on this "backbone" function, please see the *Statement on the overall impact of the project* in §II.1.1, as well as the outline of the *funding principles* in §II.3.3 of the enclosed *final report* of P24028-N18. This report is also available at

http://www.math.tu-graz.ac.at/~woess/final-report-P24028.pdf

I am again applying to the FWF for one PostDoc and one PhD position, for a period of 3,5 years.

There are at least two possible candidates for the PostDoc position, namely **Dr. Wojciech Cygan,** who already was PostDoc in my group from October 2015 to July 2017 (within the previous project as well as with temporary NAWI Graz funding). Collaboration with him, on the level of publications as well as on the one of scientific exchange was fruitful and pleasant. He stands for a more analytical approach to probabilistic parts of this proposal, where he has had an excellent training in the Wroław school, of which I have a very high esteem.

The other candidate is **Dr. Florian Lehner**, whom I had already indicated as a possible PhD candidate for the preceding project. He had gotten another PhD position at TU Graz (with my colleague H. Wallner from the Geometry Institute) but was anyway extremely autonomous in pursuing his research, so far concentrated mostly on infinite graph theory, with several interests in random structures. He was a recipient of several awards, see his web-page

http://www.florian-lehner.net/

Currently he is a PostDoc with *Agelos Georgakopoulos* at Warwick University, with a Schrödinger fellowship of the FWF which includes funding of a 1 year return phase which he plans to pursue in my working group. Thereafter I would be pleased to have him for a further period of time.

There is also at least one very good candidate for the PhD position, my new (since April 2018), excellent student **Christian Lindorfer**, currently funded for 23 months, whom I oriented towards working on self-avoiding walks.

As one can see, already the range of topics represented by these three persons is quite diverse: in fact, I like bringing together different directions within my working group, as one can see, for example, from the report on P24028. So far, this has been fruitful and, I believe, also successful.

The five topics of this new project are all concerned with "walks", random walks and related stochastic processes in the first four, and deterministic (combinatorial) with an outreach to Statistical Physics in the last one. Boundaries are present in all of them: as the state space in topic **A**, as Poisson, resp. Martin boundaries in topics **B, C, D**, and as the space of ends in topic **E**. In all cases, they are also geometric boundaries. The involved methods range from probabilistic, analytic, potential theoretic to combinatorial and formal language–theoretic.

The range of themes that we intend to investigate are the following.

- **A.** Ultrametric spaces arise as boundaries of trees. On such spaces, there is a family of natural isotropic Markov processes, induced by so-called *hierarchical Laplacians.* The plan is to combine random perturbations of those operators with random environment in the sense of randomising the underlying tree.

B. *Branching random walk* studies the evolution of a population which moves according to random walk, while increasing according to a Galton-Watson process. We intend to study the random sequence of the empirical distributions of the population and their boundary behaviour on trees and groups, in particular hyperbolic groups.

C. A basic example of a *strip complex* is "sliced hyperbolic plane", where the hyperbolic upper half plane is subdivided into isometric horizontal strips. The associated Laplace operators satisfy a Kirchhoff type condition at the gluing lines, where the space is singular. The aim is to describe the associated Martin boundary in detail.

D. In purely discrete environment, a topic related with Martin boundary concerns the general integral representation of *λ-polyharmonic functions.* Here, the plan is to address polyharmonic functions on trees also beyond the nearest neighbour case, in particular on virtually free groups, and subsequently on hyperbolic groups.

E. At first glance seemingly rather different, but part of my "panorama" in a natural way, is to study the nature of the language associated with *self-avoiding walks* on vertex-transitive graphs, in particular Cayley graphs of groups.

More details will be given below in §4. Can this project lead to some breakthrough? This is of course very hard to foresee; the appreciation of new mathematical results as being a breakthrough is related not only to their depth (maybe the depth is not even a decisive issue), but also to current trends, celebrity of the author, and so on (even randomness in the results' reception), and I have had several experiences in these respects. In any case, I believe that the research which I am proposing concerns serious mathematics leading to results that may be perceived to be important. The details of the project description in §4 constitute a honest effort to reply to those issues without unnecessary boasting.

2. FWF Doctoral program (DK) "Discrete Mathematics"

This "Doktoratskolleg" (equivalent of the German "Graduiertenkolleg") was established in 2010. It is a joint project of currently 11 mathematicians from TU Graz and the Universities of Graz (KFU) and Leoben (MUL), with substantial funding from the FWF and from TU + KFU for up to 12 years. The PhD students are internationally recruited. In the current phase, the salaries plus generous travel allowances for 9 of them are funded by FWF, 2 positions are directly funded by TU and one by KFU, and in addition there are up to 11 associated PhD students with salaries from different sources, but with the same travel allowances, etc. I am the speaker of this program. For more details, see

http://www.math.tugraz.at/discrete

Already in my previous project FWF-P24028, the project employees have interacted strongly with this doctoral program, with mutual benefits. It is clear that this will apply to the present project as well.

3. Brief overview on current work and the research group

The current & recent work within my research group is documented to a good extent in the enclosed final report of FWF P24028 and will not be repeated here in all details. Please see also the list of publications which is part of that report.

With the beginning of 2016, the two small institutes of *Optimisation and Discrete Mathematics* and *Institute of Mathematical Structure Theory* have been merged into one bigger *Institute of*

Discrete Mathematics. I was its head until recently; starting with 2018 this is my colleague **Mihyun Kang** from the sister half of the institute, who is working on random graphs and related topics, so that our interests have several meeting points. What formerly was the Institute of Mathematical Structure Theory now consists of the two working groups *Structure Theory and Stochastics,* led by myself, and *Non-commutative structures,* led by **Franz Lehner**.

Currently, the working group *Structure Theory and Stochastics* consists of the following persons. For each of them, I give a short outline of the work and some recent publications.

Wolfgang Woess, professor. My own scientific work in the last 5 years has been concentrating on the following themes, in part of course already present in the proposal for the previous project.

• The study of *context-free languages* in relation with finitely generated groups is a theme which I re-visit at irregular intervals, in part in collaboration with *Tullio Ceccherini-Silberstein* [C-W], [W3]: My last work on these topics now dates back to 2012, but I am again coming back to the theme formal languages – walks on groups, see topic **E** of this proposal.

• Since quite some time, I am collaborating with *Marc Peigné* (Tours) on *stochastic dynamical systems*, that is, iteration of random i.i.d. Lipschitz mappings on a proper metric space, see [P-W1], [P-W2]. The current interest in this topic is related with multidimensional *random walks with reflections* of different types, where the methods used in the work with Peigné prove to be of significant use. See below *(Judith Kloas).*

• Brownian motion on *strip complexes,* in particular *treebolic space* has been a long and difficult project of mine in collaboration with *Alexander Bendikov, Laurent Saloff-Coste and Maura Salvatori* [B-S-S-W1], [B-S-S-W2], [B-S-S-W3]. This work is basically concluded, but one interesting part (Martin boundary of sliced hyperbolic plane) is still to be studied, possibly in collaboration with *Sara Brofferio* besides *Alexander Bendikov.*

• Ultra-metric spaces arise as boundaries of trees. Based on an approach of *A. Bendikov,* whose clever simplicity allows to obtain a variety of new results, *isotropic jump processes* have been studied by *Bendikov, Cygan, Grigor'yan, Pittet* and others, see in particular [B-G-P-W] and [B-C-W], were I had the opportunity to contribute to this attractive research.

• The Martin boundary for random walks on trees stood at the beginning of my collaboration with *Massimo Picardello* more than 3 decades ago. Very recently, we have again begun to do joint research on boundary integral representations for λ-harmonic functions on trees [Pi-W].

• Further recent work of mine concerns a collaboration with my former PostDoc (now senior lecturer at Univ. Sydney) *James Parkinson* on regular sequences in affine buildings, work on potential theoretic issues (Riesz measures) on trees and the clarification of its relation with the hyperbolic disk, and a digression into information theory, again related with trees. See the list of publications in my CV (§13).

Daniele D'Angeli, assistant professor with habilitation. He has been one of the most successful "acquisitions" of my research group thanks to the preceding FWF project P24028. Daniele was a PhD student of *Tullio Ceccherini-Silberstein* at University of Rome-I, where he graduated in 2008. After several other international experiences, he came to Graz in 2013 within my previous FWF project and is now assistant professor in my institute. He works very successfully on various aspects of *automata groups* in the sense of *Grigorchuk, Sidki* and others, a very active topic. His multiple research contributions are featured in the enclosed *final report* of FWF P24028. Let me cite here two recent papers related with boundary actions and limit spaces of automaton groups and self-similar groups [Bo-D-N], [D-G-K-P-R]. These topics add very favourably to the other ones within the group, and in the meantime he has succeeded in obtaining his own FWF-project with one PostDoc and one PhD position, fitting nicely into the group. Daniele D'Angeli obtained his "Habilitation" in 2017.

Ecaterina Sava-Huss and **Wilfried Huss,** assistant professors. Both were my PhD students and graduated in 2010. Ecaterina started with work on Poisson boundaries of certain lamplighter graphs and groups [S], a theme which she took up again recently for another type of groups in collaboration my PhD student *Johannes Cuno* [Cu-Sa]. Wilfried started with internal diffusion limited aggregation on graphs, in particular the comb lattice, and soon started to collaborate with Ecaterina on various types of probabilistic and combinatorial aggregation models on graphs [H-S]. While Ecaterina had a PostDoc position as the coordinator of the DK "Discrete Mathematics" (see Section 2), Wilfried held PostDoc positions in Siegen, Vienna and Graz, before both of them succeeded to obtain FWF Schrödinger fellowships for a 15 months' stay at Cornell University. In the US, they also started a collaboration with *Alexander Teplyaev* and his former PhD student *Joe Chen* [C-H-S-T]. Ecaternia now has the assistant position formerly held by *Lorenz Gilch.* She and Wilfried are married, Ecaterina has recently returned from maternity leave with their second child, during which Wilfried was her substitute on the assistant position. Right now, Wilfried is on paternity leave, after which he may be looking for a job in industry.

While the above refer to (non-tenured) university positions, currently there are two PhD students and further PostDocs employed via project funding, resp. teaching assignments:

Judith Kloas was funded by my preceding FWF project P24028 until February, 2017, and now receives funding from a different source within TU Graz. She has been associated with the DK "Discrete Mathematics", benefitting from its vivid mathematical environment. She works on multidimensional random walk with reflections: an ordinary random walk evolves within the non-negative cone of $\mathbb{R}^d$, resp. $\mathbb{Z}^d$, but when a coordinate becomes negative, its sign is changed before the next step is performed. An analogous version, related with queueing theory, will set that coordinate equal to 0 when it should attain a negative value. It turned out that the multidimensional case is much, much harder (harder than expected) than in dimension one. See her papers with myself [K-W] and with *Wojciech Cygan* [Cy-K]. A side-line of her research concerns graph-theoretical questions ("distinguishing numbers") in collaboration with emeritus prof. *Wilfried Imrich* (MU Leoben) and further PhD students of the DK "Discrete Mathematics". Her PhD defense took place on March 2, 2018. Her employment will last until summer 2018, when she intends to move to Switzerland.

Gundelinde Wiegel is (since almost 2 years) a PhD student within the DK "Discrete Mathematics", her project position being sponsored by TU Graz. Formerly, she had studied at Univ. Tübingen (Germany), where the advisor of her master thesis was *Martin Zerner.* In Graz, she extended the results from that thesis and wrote a paper on random walks with random killing on $\mathbb{Z}$ and on trees [Wi]. Again, a side-line of her research concerns graph-theoretical questions in collaboration with *Wilfried Imrich* and another group of PhD students. But her main topic is now the so-called *frog model* for random walks on graphs (comparable to branching random walk, where a branching into new particles only takes place when a site is visited for the first time).

Christian Lindorfer is the other, very recent PhD student; he has already been mentioned above, currently working on parts of topic **E**..

Lorenz Gilch had an assistant professor position at my institute until 2016. Before that, he had been my PhD student. He works on random walks on discrete groups and infinite graphs, concentrating on issues such as rate of escape and entropy, employing methods from complex analysis. One body of his work concerned free products, also involving branching random walk. More recently, he has been collaborating quite intensively with my former project PostDoc *James Parkinson* (Sydney) and with *Sebastian Müller* on random walks on Fuchsian buildings and Kac-Moody groups, see e.g. Gilch, Müller and Parkinson [G-M-P]. He obtained his "Habilitation" in 2016. After his contract as an assistant professor had expired, he worked in a centre for "Big Data" research, held a substitute (full) professor position at Univ. Bayreuth in the summer semester 2017 and is now back with some part-time teaching duties at TU Graz, with another part-time position at Univ. Passau (Germany).

Wojciech Cygan should also be mentioned here. He was a PostDoc within the preceding project FWF-P24028 until recently, and is now working in Dresden with *René Schilling.* He is fully featured in the enclosed *final report* of FWF P24028, so that here I only mention his collaboration with Judith Kloas cited above [Cy-K]. He will certainly remain in contact with me and my group and possibly return for an extended period in the future, maybe within the present project.

Let me also briefly mention the second working group which goes back to the former Institute of Mathematical Structure Theory: *Non-commutative Structures.*

It is led by **Franz Lehner**, tenured associate professor position. He is working in free probability, applications in combinatorics, spectra of convolution operators, and related topics, and has an international reputation in this field.

Within this group, there are the collaborators **Dr. Amnon Rosenmann** and **Dr. Thomas Hirschler**, as well as until autumn 2017 the project PhD student **Konrad Schrempf**.

Another person who is regularly present at our institute is **Dr. Sebastian Müller**. He was a PhD student of *Nina Gantert* (Munich) and subsequently a PostDoc in my earlier FWF project P19115. After further stages, he has now a tenured Maître de Conférence position at Université Aix-Marseille. Since his family lives in Graz, he compresses his teaching duties in Marseille so that he can spend extended periods of time in Graz. He contributes substantially to the spirit and work of the research group, and in particular, he interacts with *Lorenz Gilch* and *Gundelinde Wiegel,* for whom he acts as a co-advisor.

4. Project research: details

A. Isotropic Markov processes on ultrametric spaces

An ultrametric space $(\mathcal{X}, d)$ is a metric space where the triangle inequality is replaced by $d(x,z) \le \max\{d(x,y), d(y,z)\}$. Every proper ultrametric space arises as the boundary (compact case), resp. punctured boundary (non-compact case) of a rooted tree. The most well-known example is provided by the ring of p-adic numbers $\mathbb{Q}_p$ (field, when p is prime). A variety of authors, often independently and coming from different backgrounds, have constructed and studied random processes on ultrametric spaces, resp. chosen an analytic approach to construct "Laplace" operators which are the generators of such processes. An early reference in the context of harmonic analysis is Taibleson [T]. The same spectral multipliers appeared as operators of p-adic derivative in the construction of the Vladimirov Laplacian, see Vladimirov [V] and the book by Kochubei [Ko]. Further approaches, in part more probabilistic and in part more analytic, are due to Albeverio and Karwowski [A-K], Dawson, Gorostiza and Wakolbinger [D-G-W], Rodriguez-Vega and Zúñiga-Galindo [R-Z], Pearson and Bellissard [P-B] and Kigami [K1], [K2] – to name just a few. In an important part of the literature, the operators appeared under the name *hierarchical Laplacians.*

A particularly fruitful approach via construction of *isotropic Markov semigroups* and the associated random processes goes back to Bendikov, and I was able to contribute to this in the papers of Bendikov, Grigor'yan, Pittet and Woess [B-G-P-W] and Bendikov, Cygan and Woess [B-C-W]. In more recent work, Bendikov, Grigor'yan, Molchanov and Samorodnitsky [B-G-M-S] and Bendikov and Cygan [B-C] studied random perturbations of hierarchical Laplacians and its effects on the spectrum.

Here, in addition to those random perturbations, we also want to study the effect of randomising the ultrametric space itself.

To be more precise, consider a tree with minimum degree 3 and $\mathcal{X}$ its punctured boundary with respect to one boundary point (end) ϖ. The vertices of the tree can then be aligned in "horizontal" levels H_k, the horocycles with respect to ϖ. Each vertex $v \in H_k$ then has a unique "parent" neighbour in H_{k-1} and $\deg(v) - 1$ "children" neighbours in H_{k+1}. Then $\mathcal{X}$ can be identified with the set of bi-infinite geodesics from ϖ to some other *end* of the tree, and a vertex v corresponds to the open-compact ball $B = B_v$ of all those ends where that geodesic passes through v. For the construction of a hierarchical Laplacian, we need a regular measure m supported on all of $\mathcal{X}$, with $m(\mathcal{X}) = \infty$, and a *choice function* $C : B \mapsto C(B)$ which assigns a positive number to each open-compact ball and satisfies a certain summability condition. The operator acts on locally m-integrable functions by

$$Lf(x) = \sum_{B \ni x} C(B) \left(f(x) - \frac{1}{m(B)} \int_B f \, dm \right) .$$

(The sum ranges over all balls containing x.)

We primarily consider the *homogeneous* case, where each vertex in H_k has the same number n_k of children. The tree T and the space $\mathcal{X}$ are then characterised by the sequence $(n_k)_{k\in\mathbb{Z}}$ of branching degrees, and the isometry group of $\mathcal{X}$ acts transitively. We start with a hierarchical Laplacian which is also invariant under that group: for $B = B_v$, where $v \in H_k$, the number $C(B_v) = C_k$ only depends on k. Indeed, the most prominent examples such as the Taibleson Laplacian arise in this way.

Now we consider double randomness as follows: the branching degrees (n_k) are i.i.d. integer valued random variables with values ≥ 2 and expectation p, so that $\mathcal{X}$ is a random version of $\mathbb{Q}_p$ (even when p is real), and the choice function is $C(B_v) = C_k(1 + \epsilon_v)$ for $v \in H_k$, where the C_k are deterministic, but the ϵ_v are a countable collection of (small) i.i.d. real random variables which are independent of the random sequence (n_k).

At this point we want to take up the previous work, in particular [B-G-M-S], [B-C] and [B-B-P]. In the case of two sources of randomness we are curious to see whether new phenomena emerge as compared with the random perturbations of hierarchical Laplacians on deterministic ultrametric spaces. In any case, classical limit theorems, including Law of Large Numbers and Central Limit Theorem should hold for appropriately scaled sequences of eigenvalues. Further, we want to investigate the statistics of the spectrum and convergence of related point processes. Here we aim to prove that these quantites bear a Poissonian flavour. The methods that we intend to use for these limit theorems (among other: characteristic functions, stationary processes, variance analysis) will require many variations and adjustments to fit in with the random enviroment. For Poissonian statistics we plan to exploit a mixture of information theoretic methods combined with the famous Stein approximation. Also coupling may find its application for studying the speed of convergence in the total variation distance.

Beyond the homogeneous setting, a next issue is to drop level-homogeneity of the tree and replace it by stochastic homogeneity, that is, to consider ultrametric spaces related with Galton-Watson trees.

Let me emphasize that to our best knowledge this is the first attempt to construct and investigate hierarchical Laplacians on random structures. This direction of research is in accordance with current trends in Probability Theory and at the same time it coincides with many problems in Statistical Physics. Further natural questions would concern studies on Schrödinger perturbations of hierarchical Laplacians, a further step towards the viewpoints of Statistical Physics. To sum up, in this part of the project we intend to provide an exhaustive description of a large class of random operators defined on a random ultrametric enviroment. This should lead us to new and hopefully innovative techniques as well as new and hopefully important insight.

B. Empirical distributions of random populations on graphs

Galton-Watson processes also come up in this next topic. Branching random walk (BRW) is as follows. Start with a random walk on a graph (or, in general, any irreducible denumerable Markov chain). At the beginning, we have one particle starting at an initial vertex. It is the ancestor of a population which evolves according to a Galton-Watson process and at the same time moves according the random walk: at any time, a particle which sits at a vertex (state) x will next fission into a random number of new particles according to the offspring distribution of the GW-process, and each of them moves independently to another vertex according to the transition rules of the random walk. It is known that there are a transient regime (when any finite set is almost surely evacuated after some finite time), and a weakly as well as a strongly recurrent regime (when the population comes back to any finite set with positive probability, resp. almost surely). In case of space homogeneity (invariance under a transitive group action), the weakly recurrent regime does not occur. An exposition of these facts plus basic references can be found in my textbook [W2, Ch.5].

For homogeneous trees, in the case when the GW-process does not die out, Liggett [Li] (isotropic case) and Hueter and Lalley [H-L] (general nearest neighbour walks on free groups) have studied the set of accumulation points on the boundary at infinity of the set of sites visited by the population. Other authors have studied the trace of the population, that is, the subgraph spanned by the visited sites, see e.g. Benjamini and Müller [Be-M] or Candellero and Roberts [C-R].

A new, related issue emerged after questions by my former PhD student *Elisabetta Candellero* (Warwick Univ.) in discussions with *Vadim Kaimanovich* (Univ. Ottawa) and myself: it is very natural to consider the sequence of empirical distributions of BRW: at time n, consider the random, finitely supported probability measure on the vertex set, where to each point we assign the number of particles sitting there, divided by the total number of particles – or, practically equivalent, divided by the expected population size at time n. Under quite general assumptions on the graph, resp., random walk, as well as its boundary in a suitable compactification, we can prove that the sequence of measures converges a.s. weakly to a random limit measure on the compactification. In the transient regime, it lives on the boundary. The study of the properties of this random measure is the substance of this topic, and it is natural to do this as part of the present project. Besides trees, the first objects of study should be other hyperbolic graphs.

While there is a good body of work of different type regarding the way how the population of branching random walk spreads out at infinity, the above question appears not only natural, but very challenging in the sense of extending the methods introduced Kaimanovich and Vershik [K-V] – the contribution of Kaimanovich and Woess [K-W] might also be mentioned here – to measure-valued random processes on groups and (transitive) graphs. This aspect seems not to have been considered before and may lead to substantial new insight.

Besides BRW, there are of course various other models of evolution of populations following a random walk. Among them, there is the so-called "frog model" (the name goes back to to

Durrett, see Popov [Po]), earlier called "egg model" when it first appeared in a paper of Telcs and Wormald [Te-W]: in principle, this is like BRW, where branching (fission in 2) at some site only occurs at the moment when it is visited for the first time.

Similarly, although this was mentioned in a proposal by *Ecaterina Sava-Huss,* to my knowledge so far nobody has studied branching rotor router walk, where the underlying walk is not random, but the fission according to a GW-process remains as it was: in many respects, rotor walks have turned out to be good combinatorial, deterministic approximations of random walk, see e.g. Cooper and Spencer [Co-Sp] or also recent work of my Huss and Sava [H-S]. In those cases, it is very natural to ask analogous questions to those outlined above for BRW. At the moment, this question is primarily "curiosity driven".

The main external collaboration partner for this topic is *Vadim A. Kaimanovich.*

C. The Martin boundary of "sliced" hyperbolic plane

The title refers to integral representations of positive harmonic functions of transition operators or Laplace operators over the *Martin boundary.* One of the tasks is to describe this boundary topologically, resp. geometrically in terms of the structure of the respective state space. A step below, sometimes slightly easier, is to determine the *minimal* harmonic functions. The same applies to positive eigenfunctions. The following, very analytic issue has come up in the work on Brownian motion on treebolic space [B-S-S-W2], [B-S-S-W3]: take the hyperbolic upper half plane and subdivide it into horizontal strips by the lines at heights q^n, $n \in \mathbb{Z}$, where $q > 1$. The strip S_n consists of all $z = x + i\,y$ with $q^{n-1} < y < q^n$. All strips are (hyberbolically) isometric. Then take a Laplace operator which is the ordinary hyperbolic Laplacian with a vertical drift parameter α inside each strip and a domain where along each line the two one-sided normal derivatives in the vertical direction are such that β times the lower one plus the upper one give 0. Then we want to describe the minimal positive harmonic functions for that Laplacian, or better, we want to show that the Martin boundary is the usual one, i.e., the unit circle when one passes to the disk model.

Equivalently – based on one of the harder theorems in [B-S-S-W3] – this amounts to determining the minimal harmonic functions, resp. the Martin boundary for certain random walks on the non-discrete, but non-connected closed subgroup of the affine group consisting of all matrices

$$\begin{pmatrix} q^n & b \\ 0 & 1 \end{pmatrix}, \quad n \in \mathbb{Z}\,,\ b \in \mathbb{R}\,.$$

There are quite clear ideas how the minimal harmonic functions should look like: if the *vertical drift* of the process is non-positive, then the random walk has a unique invariant Radon measure on the lower boundary line $\mathbb{R}$. This follows from work of Furstenberg [Fu], Babillot, Bougerol and Elie [B-B-E] and Brofferio [Br]. The minimal harmonic functions should then be described by the Radon-Nikodym derivatives of translates of this measure with respect to the measure itself. In addition, there must be one minimal harmonic function sitting "at ∞", which comes from the vertical projection of the random walk. In case of positive vertical drift, one should conjugate by a suitable exponential.

While the "should be"s are clear, it needs a noteworthy effort to prove the expected results, by using methods of Ancona [An] (for non-zero vertical drift, expected to be the slightly easier part) and Gouëzel [G].

This part of the project does not only concern the example case of "sliced hyperbolic plane". It also comprises applications to other matrix groups, such as the amenable Baumslag-Solitar groups and higher dimensional analogues of the above one. So far, not much work has been done for mixed non-discrete, non-connected subgroups of matrix groups, besides [B-S-S-W2] and [B-S-S-W3].

It has been a longstanding aim of mine to determine the full Martin compactification of the amenable Baumslag-Solitar groups. While I have clear ideas what it should be (the same as for Brownian motion on treebolic space), this appears to be a very hard problem. If solved, it would be at least a personal breakthrough. The research proposed here intends, among other, to get as close as possible to this aim.

External collaboration partners for this work are *Sara Brofferio* (Paris), and possibly also *Sébastien Gouëzel* (Nantes).

D. Boundary integral representation of λ-polyharmonic functions

This is also related to Martin boundary theory, but quite different, and more "discrete". For a nearest neighbour random walk P on an infinite, locally finite tree, and any real $\lambda > \rho$ (where ρ is the spectral radius), all positive λ-harmonic functions (i.e., satisfying $Ph = \lambda\, h$) have a unique integral representation with respect to the Martin kernel, which has a particularly simple form, and the boundary is of course the geometric boundary of the tree (the space of ends). This is very well understood since the seminal work of Cartier [C], who also showed that *every* (not necessarily positive) λ-harmonic function has such a representation, where in general the representing measure on the boundary is not σ-additive, but a finitely additive distribution on the space of all locally constant functions.

In my textbook [W2, Thm. 9.37], I showed how this also works for non-locally finite, countable trees. On the basis of recent discussions during work in progress of Picardello and Woess [Pi-W], I have recently been able to extend this to all complex-valued λ-harmonic functions, where λ is in the resolvent set of P, up to some degenerate values of λ. Here, the resolvent refers to P acting as a self-adjoint operator on a weighted ℓ^2-space. The weights are those which make the underlying random walk reversible, and "degenerate" refers to those λ in the resolvent set where some diagonal element of the resolvent operator *(Green function)* vanishes.

A similar (locally finite) result in a specific group-invariant case (free product of groups of order 2) was obtained by Steger, see Figà-Talamanca and Steger [F-S].

A λ-*polyharmonic* function is one which satisfies $(\lambda\, I - P)^n h = 0$. For $\lambda = 1$, polyharmonic functions on trees were studied in a long paper by Cohen, Colonna, Gowrisankaran and Singman [C-C-G-S], who derived a boundary integral representation for the case of simple random walk on a regular tree. Picardello and I found a much simpler method to obtain this for arbitrary nearest neighbour random walks on countable trees and again all complex λ as above. This is based on three facts: (1) that one may take the derivatives of the λ-Martin kernel with respect to λ, and (2), that one may exchange application of P with derivation. The n times derived Martin kernels then provide the "basis" for polyharmonic functions of order n. Then, (3), one needs the integral representation of *all* harmonic functions.

There are natural questions to be asked here. The *Riquier problem* extends the Dirichlet problem. The latter asks whether any continuous function on the boundary extends to a harmonic function which has the given boundary values. For polyharmonic functions of order n, one would like to find one where each of $(I-P)^k h$ has specified boundary values for $k = 0, \dots, n-1$. This is already quite problematic for $n = 2$ and simple random walk on homogeneous trees, see Cohen, Colonna and Singman [C-C-S], and more so in the general situation. Linked with this is the search for general results of Fatou type, extending those in the homogeneous, isotropic case of [C-C-G-S].

Beyond this, an obvious aim is to extend that study to random walks on trees which have bounded range instead of being of nearest neighbour type, and more generally to random walks on hyperbolic graphs and groups. There, (1) and (2) apear well accessible, while (3) will not be valid in general, and one needs to single out a class of "analytically" well defined λ-polyharmonic functions for which an integral representation will be valid.

Aspects of this topic seem well suitable for PhD work. In particular, one should note that while there is an enormous body of work on polyharmonic functions in the smooth setting, far beyond what can be cited here, analogous work in the setting of discrete potential theory appears to be basically limited to the few references displayed above.

The natural external collaboration partner here is *Massimo Picardello* (Rome), and again possibly also *Sébastien Gouëzel* (Nantes).

E. The language of self-avoiding walks

The last topic may appear quite different from the previous ones. Consider a locally finite graph (X, E) without loops and choose a root vertex o. Let σ_n be the number of self-avoiding walks (i.e., walks that do not self-intersect). The *connective constant* of the graph is $\sigma = \limsup \sigma_n^{1/n}$. While in itself of combinatorial nature, this object has been studied intensively in environments of Statistical Physics and Probability. I will not give a complete historical overview; see the monograph by Madras and Slade [M-S] and the lecture notes of Bauerschmidt, Duminil-Copin, Goodman and Slade [B-D-G-S]. It is in general hard to determine σ explicitly.

A *labelled graph* is obtained by first replacing each edge by two oppositely oriented edges $(e, \check{e})$ with $\check{\check{e}} = e$. Each oriented edge e then gets a label $\ell(e)$ from a finite alphabet $\mathbf{\Sigma}$ which has an involution $a \mapsto \check{a}$. The labelling must be such that if $\ell(e) = a$ then $\ell(\check{e}) = \check{a}$, and we assume that it is *deterministic:* for any vertex $x \in X$ and label $a \in \mathbf{\Sigma}$, there is at most one outgoing edge at x with label a. Typical examples are Cayley graphs of finitely generated groups: such a group is a factor of a free group, whose free generators + inverses form the alphabet $\mathbf{\Sigma}$. See e.g [C-W] for details in this spirit.

For any finite walk in the graph, the successive labels along its edges form a word over $\mathbf{\Sigma}$. Then the language of self-avoiding walks is the language of all words that are formed by reading the labels of some self-avoiding walk starting at o. What is the nature of this language within the Chomsky hierarchy of formal languages and some later extensions ?

For comparison, consider the language of all *closed* walks from o back to o (possibly self-intersecting). In important work of Muller and Schupp [Mu-S2], the class of *context-free graphs* was introduced: in brief, these are precisely the labelled graphs where the language of all words along *closed* walks is context-free. With their graph metric, they are quasi-isometric with trees, and they have finitely many cone types, i.e., isomorphism classes (as labelled graphs with boundary) of the connected components that remain after deleting a ball around o with arbitrary radius. For a Cayley graph of a group, this means that the group is virtually free, a famous result of [Mu-S1]. The analogous question regarding self-avoiding walks was inspired by a note of Zeilberger [Z] on "ladder graphs" and another, recent note of Gilch and Müller [G-M] on free products of finite graphs. If the language of self-avoiding walks is regular, resp. context free, then this yields that the generating function $\sum_n \sigma_n z^n$ is rational, resp. algebraic, so that the connective constant – the inverse of its radius of convergence – is an algebraic number.

Indeed, in famous work, Duminil-Copin and Smirnov [D-S] have even proved that the connective constant is an algebraic number, namely $\sqrt{2+\sqrt{2}}$, for the hexagonal lattice, which is not a context-free graph.

Since the first submission of this proposal, some good progress has been made: my recent master student *Christian Lindorfer* now has a PhD position with TU Graz funding for 23 months, hopefully to be extended by other sources (e.g., the present project). His work done so far leads to the following two conjectures: 1) for a vertex-transitive labelled graph, the language of self-avoiding walks is regular if and only if the graph is a free product of finite graphs. 2) The language is context-free (in fact, ultimately linear) if and only if all ends of the graph have size at most 2 in the terminology of THOMASSEN AND WOESS [Th-W]. For groups, this can be formulated in terms of free products and HNN-extensions over subgroups with cardinality 1, resp. 2. These are, in the proposer's view, exciting developments leading to interesting new insight. Boundaries are present here in form of the space of ends.

The generating function of self-avoiding walks is rational in many other cases, see e.g. ALM AND JANSON [A-J]. Thus, subseqent questions concern the relation with further classes of formal languages, e.g. those studied by CECCHERINI-SILBERSTEIN ET AL. [C-C-F-S-T]. A language-theoretic approach to self-avoiding walks appears completely new and promising, and may open a link to word processing in groups, referring to the famous book by EPSTEIN ET AL. [Ep] with that title.

In conclusion, let me explain how in my mind context free graphs and groups are related with the other topics of my research, here primarily topic **D**: The fact that Cayley graphs of virtually free groups are context-free implies that the generating function of the return probabilities to the identity are algebraic for any finite range random walk on such a group. I have explained this in an old note [W1], and it is related with work of AOMOTO [Ao] and FIGÀ-TALAMANCA AND STEGER [F-S] on the spectral theory and harmonic analysis associated with such random walks.

The natural collaboration partner for this topic is *Tullio Ceccherini-Silberstein.* Local exchange with *Daniele D'Angeli* and *Amnon Rosenmann* may also be fruitful.

Once again, in the same spirit as stated in my previous project proposals, the research topics outlined here may be slightly more than a group of two or three young researchers plus external partners collaborating with me may be able to solve completely within three and a half years. Let me clarify that the above is a reservoir of interesting questions; already in the past projects, practically all issues raised in the proposals were addressed successfully by members of my research group & myself and partners: as before, the present project should have a significant impact on the research group on the whole, going beyond the collaborators which are funded directly by the project. And, also of importance, the young project collaborators will achieve a view going beyond the focused issues of their respective individual research. As a matter of fact, as one can see from my previous projects, rather than primarily aiming at many joint papers of myself with the project collaborators, an important goal is to provide them with research problems and an environment in which they can sharpen their autonomy as well as widen their panorama. This has, of course, a positive impact on my own work.

5. ETHICAL ASPECTS

The proposed project does not involve any kind of problematic ethical issues.

6. PROJECT DURATION

The project should have a duration of 3,5 years and start in 2018/19.

Indeed, this is the last stand alone project application of my carreer: my active service is scheduled to terminate on September 30, 2022. As professor emeritus, I will be entitled to continue to conduct research even after that date. While I am committed to maintain my research activities beyond 2022, it appears reasonable to set the above formal limit for the project.

7. Project personnel

I am applying for one PostDoc position for 3,5 years and one PhD position for 2 years. The reason for this subdivision is that currently I have two open years for PhD funding from a different source, while in these times it is harder for good young PhDs to find suitable PostDoc positions.

As mentioned already in the introduction, good candidates for PostDoc are **Dr. Wojciech Cygan** and **Dr. Florian Lehner**. For the PhD position, I was right to place my hopes on the very recent PhD student **Christian Lindorfer**, who has had a very good start, and I do hope that I can extend his funding beyond the current 23 months from a suitable source.

None of the named three persons is female, but (1) please have a look at my records to see that more than 50% of my PhD students have been female, and (2) in any case, I always have and will continue to make a broader search for good candidates in any case, e.g. by advertising on several email announcement lists and contacting colleagues abroad.

Again, a main educational aspect of the present project is its tight link with the FWF doctoral program (DK) “Discrete Mathematics”, of which I am the speaker.

And again, as previously, please note that the topics presented here have a strong intersection with, but go beyond the ordinary scope of discrete mathematics & probability alone. My ongoing interest is to broaden the research which I had started many years ago within the area “random walks on infinite graphs and groups”, and to do research at the intersection of different mathematical disciplines. In this spirit, it is my explicit intention to unite several, different but connected research topics under the “roof” of one project.

8. International collaboration

As in all preceding projects, my multiple international contacts will have an ongoing benefit for the research group, and in particular, for the project staff.

As in previous years, there are several ways of funding for inviting foreign partners to Graz for stays of one week up to 2 months:

(1) visiting professorships at TU Graz (with not too attractive payment, but sufficient to cover costs);

(2) short visits in Graz can in part be funded by the budget for colloquium speakers (“Gastvortragende”) and visiting scientists of the faculty at TU Graz;

(3) there is the possibility of exchange of teaching personnel within the EU Erasmus programme;

(4) visitors within the already mentioned FWF doctoral program “Discrete Mathematics”. (One can expect that the project PhD can become an associated DK student.)

My strategy has always been to use all possible funding sources optimally. In particular, the primary use of the project funding always has to be the funding for the project personnel, while so far only very rarely did I use project money for project related conference participations of myself.

Specific international collaborations on the basis of visits of a few weeks in either direction (guests coming to Graz, or project collaborators from Graz going abroad) are again planned for this project. Several international partners were already named above in Section 4. In particular, the following persons from abroad should be part of an exchange within the work on the topics proposed above.

(I) Alexander Bendikov, Professor at Wroław University, is a very reliable expert in potential theory and a strongly analytic approach to probabilistic issues, with continuously increasing interest in random walk questions. In the last years, we have collaborated on two different topics: (a) the construction of diffusion operators on "strip complexes", in particular treebolic spaces, and the analysis of the associated Brownian motion and potential theory on them, and (b) isotropic Markov processes on ultrametric spaces, the related infinitesimal generators ("hierarchical Laplacian"), its relation with random walks on trees, and the asymptotics of transition probabilities. Alexander Bendikov has visited TU Graz several times, we share 5 joint publications, and I was external referee & examiner for the PhD defense of Bendikov's PhD student Wojciech Cygan [Cy], who then came to Graz as a PostDoc. Bendikov's expertise for the present project regards, in particular, topic **A**.

The interaction with Wrocław University is not only limited to Bendikov and Cygan, but also includes the group of **Ewa Damek** and her colleagues, in particular, **Dariusz Buraczewski**.

(II) Sara Brofferio, Maître de conférences at Unv. Paris-Sud with habilitation, was a student of mine in Milano in the 1990ies. I was the advisor of her master thesis ("tesi di laurea"). Thereafter she went to Paris, where she became a PhD student of the late Prof. *Martine Babillot.* After that, she came to Graz in the earlier years of this millennium with a EU Curie fellowship before settling in Paris. While not a high frequency publisher, she has a high capacity of doing deep and beautiful mathematical research. We share 4 publications. Regarding the present project, she is a natural collaboration partner for topic **D**, which is at the centre of our common interests.

(III) Tullio Ceccherini-Silberstein started with C^*-theory, then became more interested in harmonic analysis and Gelfand pairs associated with discrete structures, and cellular automata under a viewpoint of dynamical systems theory. Furthermore, he has been successfully working on combinatorial group theory, growth of groups, and the interplay between groups and formal languages. On this last issue, we have 3 papers in collaboration. Tullio has authored several research monographs and edited proceedings volumes. He has an amazing number of good international contacts with outstanding mathematicians from many countries. (Only in his own country he is being treated far below his merits)Already in the past, Ceccherini-Silberstein has visited Graz several times. His invaluable input will be important for topic **E**.

(IV) Sébastien Gouëzel is an outstanding younger "star" in the field of random walks on discrete groups. He comes from the famous probability & ergodic theory school of Rennes and is now Directeur de recherche CNRS at Université de Nantes. Within a few years, he has completely solved the question of the exact asymptotics of return probabilities for random walks on hyperbolic groups, provided significant new insight concerning the Martin boundary, as well as the asymptotic entropy of those walks, to mention just a few of his important contributions to different fields of current research. We know each other and had some discussions; he visited Graz for a week some years ago. I hope that he will be willing to come again to Graz and / or host some of my collaborators for shorter visits.

(V) Vadim A. Kaimanovich is since many years a partner of the research activities of myself and my group of younger collaborators. Several years ago, he moved from Bremen to the University of Ottawa, Canada. He is the leading expert on Poisson boundary theory and related subjects. His recent work comprises, among other, his measure-theoretic approach to random graphs and collaboration with *Anna Erschler* (another expert in Poisson boundaries and further topics at the interface of geometric group theory and random walks). Also, recently (after more than 10 years) he finally wrote up and published substantial work on Poisson boundaries of random walks on Thompson's group F – this is related with the very "hot" question of amenability of that group. *Agelos Georgakopoulos* spent some time in Ottawa in between two PostDoc stays in my group, and recently, my PhD student *Johannes Cuno* spent a year in Ottawa after graduating. Kaimanovich and I have 4 joint papers. As outlined, topic **C** is the outcome of discussions with him, and he will be involved in this work.

(VI) Massimo Picardello is professor at Università di Roma - Tor Vergata. He is a harmonic analyst. I came in contact with him and Prof. *Alessandro Figà-Talamanca* in the 1980ies via their work on harmonic analysis on free groups. Since 1984 (when I spent a year in Rome), we established an intensive collaboration on Martin boundary theory on trees and related issues. Since then, Picardello's main research topics have been analysis, integral geometry and potential theory on trees. From 1985 to 1994, we published 10 joint papers. In a very recent visit of Massimo to Graz, our collaboration revived very fruitfully. In the present project, this has lead to Topic **D**, and future collaboration should have a positive impact on this project.

(VII) Laurent Saloff-Coste, professor at Cornell University, is a prominent and leading figure in current research on random walks on groups, finite Markov chains as well as analysis and potential theory on manifolds, groups and other structures. He has been member of the editorial boards of several top journals, and he is part of the advisory board of the DK "Discrete Mathematics" mentioned above. We have 7 joint papers. Saloff-Coste has come to TU Graz as a visiting professor in 2003. We meet regularly on different occasions. After his long service as the chairman of the Mathematics Department at Cornell University, it has become harder to get him to Graz, but I hope this will be possible in one of the next years. Sending my collaborators to Cornell is highly profitable for them. My PhD student *Johannes Cuno* spent a semester there, and both *Wilfried Huss* and *Ecaterina Sava-Huss* were there for 15 months with an FWF-Schrödinger fellowship, all of them not directly working with, but close to Saloff-Coste. His input will be precious for the entire project.

9. Research plan

As stated in the previous project proposal, in mathematical research it is not very common to follow a fixed timetable such as, e.g., in laboratory experiments. But I expect to proceed according to the following, rather logical order.

In the first one and a half or two years, the postdoc research assistant (who could be Wojciech Cygan) will collaborate with myself on topic **A**. Alexander Bendikov will be a very natural external collaboration partner.

The 2 years' PhD position should be filled in the middle of year 2. In case the future PhD student is going to be Ch. Lindorfer: he has already started to work on Topic **E** – at the beginning with funding from a different source which is available at TU Graz, subsequently hopefully within this project.

Soon after that, Florian Lehner will be back to Graz within the return phase of his Schrödinger fellowship: he would be very suitable for joining in this research and project after that return phase.

Topic **D** would also be well suited for PhD work, and Massimo Picardello will be involved in this phase. The choice which one among **D** and **E** to assign to a doctoral student should depend on the specific abilities and preferences of the chosen one among the candidates for the PhD position.

In the subsequent period, work on Topics **B** and **C** should be attacked by all project team members, interacting with Vadim Kaimanovich, Sara Brofferio and possibly Sébastien Gouëzel, of course including my own input and participation.

In particular, as implied by many previous experiences, the timetable for the work of the PhD student – possibly Ch. Lindorfer, and if so then officially within this project only for his last 2 years – is already following the logical path, by having gotten closely acquainted with the subject. The first 3–4 semesters are also dedicated to fulfilment of the curricular requirements (=courses and seminars) of PhD studies, while in the 2nd and 3rd year the PhD student should get used to attending conferences and – as soon as possible – summer schools and related activities, thus setting foot in the international mathematical community. Towards the end of 2019, or even earlier, one may expect a first publication.

10. Importance, impact, dissemintation

Regarding the impact, please see once more the *Statement on the overall impact of the project* in the enclosed *final report* of the preceding project P24028-N18. As in the past, the proposed project will be the backbone of my working group "Structure Theory and Stochastics" at the Institute of Discrete Mathematics of TU Graz, to a much larger extent than the sole employment of a PostDoc and a PhD student. Repeating a previous statement, it will allow this group to maintain its good reputation, which we managed to acquire in the last 15 years, owing greatly to the successive FWF projects, which contributed to the scientific development of a good number of young persons.

Closely linked with the FWF doctoral program (DK) "Discrete Mathematics", it will again be an important part of the good standing and visibility of Mathematics in the universities located in the Austrian region of Styria, primarily in Graz. In particular, the link with the DK is very relevant for the educational aspect of the project work.

As before, dissemination of project results will follow the well-established and efficient habits of the international mathematical community. Postings on *arXiv* guarantee the (green) open access requirements of the FWF in any case and reach the target audience very effectively, well before papers appear in a peer-reviewed journal. Conference talks and poster presentations are a regular part of our activities. Support by the FWF is of course mentioned on all those occasions. The maintenance of web-pages of the project (resp., its publications), of the team members, and also of the institute complete the picture.

11. Requested funding

The requested funding is primarily for a PostDoc fellowship for 3,5 years and for a PhD fellowship for two years. The salary costs in the below table are taken from the FWF website.

The team members should get opportunities to visit the above-mentioned international partners. I am estimating the costs for an overseas flight at 800 € and the daily costs during the stay at 75 €. Travel costs and daily costs within Europe are estimated at 380 €, resp. 75 €, so that for a trip of 10 days amounts to an estimate of 1.550 Euro (overseas), resp. 1.130 € (Europe).

Again, I also plan to bring the international project partners of Section 8 to Graz, using project funds for periods of 8–12 days for three of them. For participation in conferences by the project PostDoc and PhD, the same estimate leads to 830 € for 6 days. Participation in summer schools or similar events by the PhD can most likely be funded in good part from different sources.

A visit of Kaimanovich from overseas should take place on an occasion when he is already in Europe. Additional stays should be financed by other sources, following my above-mentioned principle to use all available funding possibilities optimally. Further exchange visits, e.g. involving Sébastien Gouëzel (Nantes) or Massimo Picardello (Rome) should use different funding sources.

The funding of the visitors' stays is computed by taking the sum of the room charge for the most convenient nearby hotel ("Hotel Mobene"), which currently charges 65 €per night (for TU Graz institutes), plus the official daily allowance of 26,40 €according to the "Reisegebührenvorschrift des Bundes" (federal reimbursement rule), amounting to a total of 91,40 Euro per day. I am estimating their average travel costs only within Europe at an amount of approximately 380 €. Thus, a visit of 10 days amounts to estimated costs of 1.294 €.

The timetable outlined in §9 and inherent in the following table may of course vary according to the definitive project personnel.

	1st year	2nd year	3rd year	3,5th year
PostDoc salaries	66.070,00	66.070,00	66.070,00	33.035,00
PhD salary		18.840,00	37.680,00	18.840,00
Bendikov to Graz	1.294,00			
Kaimanovich to Graz		1.294,00		
Brofferio to Graz			1.294,00	
PostDoc to Ottawa		1.550,00		1.550,00
PostDoc to Paris	1.130,00			
PhD to Rome		1.130,00		1.130,00
Conferences	830,00	830,00	1.660,00	1.660,00
total (€)	69.324,00	89.714,00	106.704,00	56.215,00

To the sum, by the FWF regulations, 5% unspecified general project costs are added.

Grand total: 338.054,85 €

Wolfgang Woess, Graz, May 2018.

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[K-W] Kaimanovich, V. A., and Woess, W.: *Boundary and entropy of space homogeneous Markov chains,* Ann. Probab. **30** (2002) 323–363.

[K1] Kigami, J.: *Dirichlet forms and associated heat kernels on the Cantor set induced by random walks on trees,* Advances in Mathematics **225** (2010) 2674–2730.

[K2] Kigami, J.: *Transitions on a noncompact Cantor set and random walks on its defining tree,* Ann. Inst. Henri Poincaré Probab. Stat. **49** (2013) 1090–1129.

[K-W] Kloas, J., and Woess, W.: *Multidimensional random walk with reflections,* Stochastic Processes and Appl., to appear.

[Ko] Kochubei, A. N.: *Pseudo-differential Equations and Stochastics over non-Archimedian Fields.* Monographs and Textbooks in Pure and Applied Mathematics **244**, Marcel Dekker, New York, 2001.

[Li] Liggett, T.: *Branching random walks and contact processes on homogeneous trees,* Probability Theory and Related Fields 106, 495–519 (1996).

[M-S] Madras, N., and Slade, G.: *The Self-avoiding Walk,* Probability and its Applications, Birkhäuser, Boston, MA, 1993.

[Mu-S1] Muller, D. E., and Schupp, P. E.: *Groups, the theory of ends and context-free languages,* J. Comput. System Sc. **26** (1983) 295–310.

[Mu-S2] Muller, D. E., and Schupp, P. E.: *The theory of ends, pushdown automata, and second-order logic,* Theoret. Comput. Sci. **37** (1985) 51–75.

[P-B] Pearson, J., and Bellisard, J.: *Noncommutative riemannian geometry and diffusion on ultrametric Cantor sets,* J. Noncommut. Geometry **3** (2009) 447–480.

[P-W1] Peigné, M., and Woess, W: *Stochastic dynamical systems with weak contractivity properties, I. Strong and local contractivity. With a chapter featuring results of Martin Benda,* Colloquium Math. **125** (2011) 31–54.

[P-W2] Peigné, M., and Woess, W: *Stochastic dynamical systems with weak contractivity properties, II. Iteration of Lipschitz mappings,* Colloquium Math. **125** (2011) 55–81.

[Pi-W] Picardello, M. A., and Woess, W.: *On boundary representations of λ-harmonic and polyharmonic functions on trees,* in preparation.

[Po] Popov, S. Yu.: *Frogs and some other interacting random walks models,* in *Discrete Random Walks,* 277–288, Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2003.

[S] Sava, E.: *A note on the Poisson boundary of lamplighter random walks,* Monatshefte Math. **159** (2010) 379–396.

[R-Z] Rodriguez-Vega, J. J., and Zúñiga-Galindo, W. A.: *Taibleson operators, p-adic parabolic equations and ultrametric diffusion,* Pacific J. Math. **237** (2008) 327–347.

[T] Taibleson, M. H.: *Fourier Analysis on Local Fields,* Princeton Univ. Press, 1975.

[Te-W] Telcs, A., and Wormald, N.: *Branching and tree indexed random walks on fractals,* J. Appl. Probab. **36** (1999) 999–1011.

[Th-W] Thomassen, C., and Woess, W.: *Vertex-transitive graphs and accessibility,* J. Combinatorial Th., Ser. B. **58** (1993) 248–268.

[V] Vladimirov, V. S.: *Generalized functions over the field of p-adic numbers.* (Russian) Uspekhi Mat. Nauk **43** (1988) 17–53. English translation in Russian Math. Surveys **43** (1988) 19–64

[Wi] Wiegel, G: *The relation between quenched and annealed Lyapunov exponents in random potential on trees,* Stochastic Processes and Appl., to appear.

[W1] Woess, W.: *Context-free languages and random walks on groups,* Discrete Math. **67** (1987) 81–87.

[W2] Woess, W.: *Denumerable Markov Chains - Generating Functions, Boundary Theory, Random Walks on Trees,* EMS Textbooks in Mathematics, European Mathematical Society Publishing House, 2009.

[W3] Woess, W.: *Context-free pairs of groups. II - Cuts, tree sets, and random walks,* Discrete Mathematics **312** (2012) 157–173.

[Z] Zeilberger, D.: *Self-avoiding walks, the language of science, and Fibonacci numbers,* J. Statist. Plann. Inference **54** (1996) 135–138.

13. Curriculum vitae and publications of Wolfgang Woess

http://www.math.tu-graz.ac.at/~woess/

1) Research topics

Random walks on infinite graphs and groups — discrete potential theory — boundary theory and harmonic functions — random processes and harmonic functions on strip complexes and ultrametric spaces — stochastic dynamical systems in discrete time — structure of infinite graphs, groups, and formal languages

2) Education and employment

Born in Vienna (Austria) 23 July 1954.

Studies of Mathematics at the Technical University of Vienna and the Universities of Munich and Salzburg.

1978: Diploma (≡ Master) degree in Mathematics at the Technical University of Vienna.

1980: Ph. D. in Matematics at the University of Salzburg (supervisor: Peter Gerl).

1980–81: Research grant at the University of Salzburg.

1982–88: University assistant (Assistant professor) at the Institute of Mathematics and Applied Geometry of the Montanuniversität Leoben (Austria).

1984–85: on leave, research fellowship (sponsored by the Italian CNR) at the Department of Mathematics of the University of Rome (research group of A. Figà-Talamanca).

1985: "Habilitation" in Mathematics at the University of Salzburg.

1987: Biennial prize (Förderungspreis) of the Austrian Mathematical Society.

1988–1994: Associate professor for Mathematical Analysis at the University of Milan (Italy).

1994–1999: Full professor of Probability and Statistics at the University of Milan (Italy) and (1998–1999) at the second University of Milan.

Since September 1999: Full professor of Mathematics at the Technical University of Graz.

Throughout the years: visiting professorships at various universities, e.g. Cornell Univ., Institut Poincaré, Univ. Sydney, Univ. Rome-I, Univ. Tours, ...

3) Organisational duties and activities

1990–1999: Associate Editor of the journal "Circuits, Systems and Signal Processing"

2000–2003: secretary of the Austrian Mathematical Society (ÖMG)

2005– : Head of the committee for doctoral studies at TU Graz.

2007–2013: Member of the Senate of TU Gra z(2013 – : substitute member).

2009–2018: Chairman of the regional section of Styria of the Austrian Mathematical Society.

Throughout the years: (co-)organisation of several conferences, e.g. at Schrödinger Institute (Vienna), Cortona (Italy), Mainz (Germany), TU Graz, Judenburg (Austria), Rome, ...

4) Most important externally funded projects

A. Asymptotic Properties of Random Walks on Graphs
FWF (Austrian Science Fund) Projekt Nr. P15577-N05, 1.10.2002 – 15.7.2006

B. Internal Diffusion Limited Aggregation on Non-Homogeneous Structures
EU Marie Curie PostDoc fellowship HPMF-CT-2002-02137, 1.3.2003 – 31.8.2004
Fellow: Dr. Sara Brofferio

C. Random Walks, Random Configurations, and Horocyclic Products
FWF (Austrian Science Fund) Projekt Nr. P19115-N18, 1.10.2006 – 31.3.2011

D. Asymptotic Behaviour of Random Walks and Branching Random Walks on Free Products
Doc fForte PhD grant of the Austrian Academy of Sciences, 1.4.2010 – 1.4.2012
Grant recipient: Elisabetta Candellero, *Grant supervisor:* Prof. Wolfgang Woess

E. Hyperbolic Structures in Stochastics, Graph Theory, and Topology
FWF (Austrian Science Fund) Projekt Nr. P24028-N18, 15.5 – 14.9.2012 and 15.4.2013 – 14.11.2017

F. Doctoral Program "Discrete Mathematics" FWF Doktoratskolleg Nr. W1230-N13
1.5.2010 – 31.12.2014 (phase 1) & 1.1.2015–31.12.2018 (phase 2) & forthcoming evaluation for phase 3. *DK speaker:* Prof. Wolfgang Woess
11 faculty members from TU Graz and the Universities of Graz and Leoben, 20 PhD students.

In addition, there were an FWF Schrödinger return fellowship (2001–2004), an FWF individual fellowship (2006–2009) and two DFG fellowships (2007–2009 and 2009–2010), for all of which Woess was the grant supervisor.

5) Publications since 2011

Peer reviewed research papers

[1] A. Bendikov, L. Saloff-Coste, M. Salvatori and W. Woess: *The heat semigroup and Brownian motion on strip complexes,* Advances in Mathematics **226** (2011) 992–1055.

[2] M. Peigné and W. Woess: *Stochastic dynamical systems with weak contractivity properties, I. Strong and local contractivity. With a chapter featuring results of Martin Benda,* Colloquium Math. **125** (2011) 31–54.

[3] M. Peigné and W. Woess: *Stochastic dynamical systems with weak contractivity properties, II. Iteration of Lipschitz mappings,* Colloquium Math. **125** (2011) 55–81.

[4] S. Brofferio, M. Salvatori and W. Woess: *Brownian motion and harmonic functions on Sol(p,q),* Internat. Math. Research Notes (IMRN) **22** (2012) 5182–5218.

[5] T. Ceccherini-Silberstein and W. Woess: *Context-free pairs of groups. I - Context-free pairs and graphs,* European J. Combinatorics **33** (2012) 1449–1466.

[6] W. Woess: *Context-free pairs of groups. II - Cuts, tree sets, and random walks,* Discrete Mathematics **312** (2012) 157–173.

[7] A. Bendikov, A. Grigor'yan, Ch. Pittet and W. Woess: *Isotropic Markov semigroups on ultrametric spaces,* Uspekhi Mat. Nauk **69** (2014) 3–102. English original in Russian Math. Surveys **69** (2014) 589–680.

[8] A. Bendikov, L. Saloff-Coste, M. Salvatori and W. Woess: *Brownian motion on treebolic space: escape to infinity,* Revista Matematica Iberoamericana **31** (2015) 935–976.

[9] J. Parkinson and W. Woess: *Regular sequences and random walks in affine buildings,* Ann. Institut Fourier (Grenoble) **65** (2015) 675–707.

[10] T. Boiko and W. Woess: *Moments of Riesz measures on Poincaré disk and homogeneous tree – a comparative study,* Expositiones Math. **33** (2015) 353–374.

[11] A. Bendikov, L. Saloff-Coste, M. Salvatori and W. Woess: *Brownian motion on treebolic space: positive harmonic functions,* Ann. Institut Fourier (Grenoble) **66** (2016) 1691–1731.

Peer reviewed research papers to appear, resp. under revision

[12] A. Bendikov, W. Cygan and W. Woess: *Oscillating heat kernels on ultrametric spaces,* Journal of Spectral Theory, to appear.

[13] J. Kloas and W. Woess: *Multidimensional random walk with reflections,* Stochastic Processes Appl., to appear.

[14] T. Hirschler and W. Woess: *Comparing entropy rates on finite and infinite rooted trees,* IEEE Trans. Information Theory, to appear.

Research papers submitted for peer review

[15] M. A. Picardello and W. Woess: *Boundary representations of λ-harmonic and polyharmonic functions on trees,* submitted.

[16] M. A. Picardello and W. Woess: *Multiple boundary representations of λ-harmonic functions on trees,* submitted.

Other publications

[17] W. Woess: *On the duality between jump processes on ultrametric spaces and random walks on trees,* unpublished manuscript (2012); arXiv:1211.7216. (This material has been merged into [7], but this version is different in a few points.)

[18] W. Woess: *What is a horocyclic product, and how is it related to lamplighters ?,* Internat. Math. Nachrichten of the Austrian Math. Soc. **224** (2013) 1–27. (An expository article.)

6) Ten publications considered most important, prior to 2011

(In order to avoid repetitions from the above list, this only contains publications prior to 2011!)

[A] W. Woess: *Nearest neighbour random walks on free products of discrete groups,* Bollettino Unione Mat. Italiana **5-B** (1986) 691–982.

[B] P. M. Soardi and W. Woess: *Amenability, unimodularity, and the spectral radius of random walks on infinite graphs,* Math. Zeitschrift **205** (1990) 471–486.

[C] C. Thomassen and W. Woess: *Vertex-transitive graphs and accessibility,* J. Combinatorial Th., Ser. B. **58** (1993) 248–268.

[D] D. I. Cartwright, V. A. Kaimanovich and W. Woess: *Random walks on the affine group of local fields and of homogeneous trees,* Ann. Institut Fourier (Grenoble) **44** (1994) 1243–1288.

[E] W. Woess: *Random Walks on Infinite Graphs and Groups,* Cambridge Tracts in Mathematics **138**, Cambridge University Press, 334+xi pages, 2000. Paperback re-edition 2008.

[F] V. A. Kaimanovich and W. Woess: *Boundary and entropy of space homogeneous Markov chains,* Ann. Probab. **30** (2002) 323–363.

[G] S. Brofferio and W. Woess: *Green kernel estimates and the full Martin boundary for random walks on lamplighter groups and Diestel-Leader graphs,* Annales Inst. H. Poincaré (Prob. & Stat.) **41** (2005) 1101–1123.

[H] F. Lehner, M. Neuhauser and W. Woess: *On the spectrum of lamplighter groups and percolation clusters,* Mathematische Annalen **342** (2008) 69–89.

[I] L. Bartholdi, M. Neuhauser and W. Woess: *Horocyclic products of trees,* J. European Math. Society **10** (2008) 771–816.

[J] W. Woess: *Denumerable Markov Chains - Generating Functions, Boundary Theory, Random Walks on Trees,* EMS Textbooks in Mathematics, European Mathematical Society Publishing House, xviii+351 pages, 2009.

14. Curriculum vitae and publications of Florian Lehner

http://www.florian-lehner.net/

Dr. Florian Lehner is a possible candidate for the PostDoc position. Please do not confound him with Assoc. Prof. Dr. Franz Lehner.

1) Research topics

Infinite graphs

Games on graphs

Connections between groups and graphs

Random structures and probabilistic methods

2) Education and Studies

born 23.6.1984 in Linz (Upper Austria). School eduaction in Linz.

Diploma Programme: Electrical Engineering, Graz University of Technology, 2004–2005.
Changed his major to mathematics after one year.

Bachelor Programme: Technical Mathematics, Graz University of Technology, 2005–2008.

Master Programme: Mathematical Computer Science, Graz University of Technology, 2008–2011

PhD: Mathematics and Scientific Computing, Graz University of Technology, 2011–2014.
PhD thesis: *Symmetry breaking in graphs and groups*
Associated member of the DK Discrete Mathematics.
Graduation with highest distinctions "sub auspiciis Praesidentis"

3) Employment

University assistant (pre PhD), Graz University of Technology, Institute of Geometry, 2011–2015.

University assistant (PostDoc), University of Hamburg, Department of Mathematics, 2015–2017.

Research fellow (FWF Schrödinger fellowship), University of Warwick, Department of Mathematics, 2017–

4) Awards

CMSA Student Prize 2013, awarded for the best student presentation at the 37ACCMCC conference in Perth.

Promotio sub auspiciis praesidentis (PhD graduation under the auspices of the Austrian president) 2015.
(Highest level of distinction for study performance in Austria, awarded for passing all final exams starting from secondary education with distinction.)

Exzellenzstipendium (excellence grant) 2015, awarded by the Austrian ministry of science for outstanding study performance.

Studienpreis der ÖMG 2015, awarded by the Austrian Mathematical Society for outstanding dissertations in mathematics, only one award in 2015.

5) Publications

Peer reviewed research papers

[1] F. Lehner: *Random colorings and automorphism breaking in locally finite graphs,* Combinatorics, Probability and Computing **22** (2013) 885–909.

[2] F. Lehner: *On spanning tree packings of highly edge connected graphs,* Journal of Combinatorial Theory B **105** (2014) 93–126.

[3] J. Cuno, W. Imrich and F. Lehner: *Distinguishing graphs with infinite motion and nonlinear growth,* Ars Mathematica Contemporanea **7** (2014) 201–213.

[4] W. Imrich, R. Kalinowski, F. Lehner and M. Pilśniak: *Endomorphism breaking in graphs,* Electronic Journal of Combinatorics **21** (2014), research paper ♯16.

[5] F. Lehner and R. G. Möller: *Local finiteness, distinguishing numbers and Tucker's conjecture,* Electronic Journal of Combinatorics, **22** (2015), research paper ♯19.

[6] F. Lehner: *Distinguishing graphs with intermediate growth,* Combinatorica **36** (2016) 333–347.

[7] M. Hamann, F. Lehner and J. Pott: *Etending cycles locally to Hamilton cycles,* Electronic Journal of Combinatorics, **23** (2016), research paper ♯49.

[8] T. Boiko, J. Cuno, W. Imrich, F. Lehner and C. E. van de Woestijne: *The Cartesian product of graphs with loops,* Ars Mathematica Contemporanea **11** (2016) 1–9.

[9] M. Hellmuth and F. Lehner: *Fast factorization of Cartesian products of (directed) hypergraphs,* Theoretical Computer Science **615** (2016) 1–11.

[10] F. Lehner: *Pursuit evasion on infinite graphs,* Theoretical Computer Science A **655** (2016) 30–40.

Peer reviewed research papers to appear

[11] F. Lehner and S. Wagner: *Maximising the number of independent sets in connected graphs,* Graphs and Combinatorics, to appear.

[12] F. Lehner: *Breaking graph symmetries by edge colourings,* Journal of Combinatorial Theory B, to appear.

[13] N. Bowler, J. Erde, P. Heinig, F. Lehner and M. Pitz: *A counterexample to the reconstruction conjecture for locally finite trees,* Bulletin of the London Mathematical Society, to appear.

Research papers submitted for peer review

[14] F. Lehner and C. Temmel: *Clique trees of infinite locally finite chordal graphs,* preprint (2013).

[15] N. Bowler, J. Erde, P. Heinig, F. Lehner and M. Pitz: *Non-reconstructible locally finite graphs,* preprint (2016).

[16] J. Carmesin, F. Lehner and R. G. Möller: *On tree-decompositions of one-ended graphs,* preprint (2017).

15. Curriculum vitae and publications of Wojciech Cygan

http://www.math.uni.wroc.pl/~cygan/

Dr. Wojciech Cygan is a possible candidate for the PostDoc position.

1) Research topics

Random walks and discrete subordination

Probability and analysis on ultrametric spaces

Convolution semigroups of probability measures on $\mathbb{R}^d$

Properties of paths of Lévy processes in $\mathbb{R}^d$

2) Education and Studies

born 29.12.1986 in Opole (Poland). School education in Opole.

2006–2011 studies of Mathematics at Wrocław University (Poland) (diploma = master 2011).

2011–2015 PhD studies at Wrocław University (advisor: A. Bendikov).

PhD thesis: *Asymptotic properties of random walks on the unit grid*

2015 PhD with distinction.

3) Employment

since October 2015 assistant professor (adiunkt) at the Institute of Mathematics of Wrocław University, on leave:

2015-2017 research fellowship (Post-doc) at the Institute of Discrete Mathematics of Graz University of Technology (research group of W. Woess, funded by FWF, NAWI Graz).

since October 2017 research assistant (wissenschaftlicher Mitarbeiter) at the Institute of Mathematical Stochastics of TU Dresden (group of R. Schilling).

4) Awards

The master thesis *Asymptotic of the Green function for subordinated random walk* was awarded by the Polish Mathematical Society in the national competition for the best student work in applied mathematics and probability theory.

June 2013: one month scholarship sponsored by German Academic Exchange Service (DAAD).

2012–2014: scholarship for outstanding PhD students, Wrocław University.

5) Publications

Peer reviewed research papers

[1] A. Bendikov and W. Cygan: *Alpha-stable random walk has massive thorns,* Colloquium. Math. **138** (2015) 105–129.

[2] A. Bendikov and W. Cygan: *On massive sets for subordinated random walks,* Math. Nachr. **288** (2015) 841–853.

[3] W. Cygan and T. Grzywny: *Heat content for convolution semigroups,* J. Math. Anal. Appl. **446** (2017) 1393–1414

[4] W. Cygan, W., T. Grzywny and B. Trojan: *Asymptotic behaviour of densities of unimodal convolution semigroups,* Trans. Amer. Math. Soc. **369** (2017) 5623–5644.

Peer reviewed research papers to appear

[5] A. Bendikov, W. Cygan and B. Trojan: *Limit theorems for random walks*, Stochastic Processes Appl., to appear.

[6] A. Bendikov, W. Cygan and W. Woess: *Oscillating heat kernels on ultrametric spaces,* Journal of Spectral Theory, to appear.

[7] A. Bendikov and W. Cygan: *On the rate of convergence in the central limit theorem for the hierarchical Laplacian,* ESAIM: Probability and Statistics, to appear.

Research papers submitted for peer review

[8] W. Cygan and T. Grzywny: *A note on the generalized heat content for Lévy processes,* preprint, arXiv:1703.10790

[9] W. Cygan and J. Kloas: *On recurrence of the multidimensional Lindley process,* preprint, arXiv:1707.01714

16. Curriculum vitae of Christian Lindorfer

Dipl.-Ing. Christian Lindorfer is a possible candidate for the PhD position.

1) Education and Studies

born 29.6.1992 in Vöcklabruck (Austria). School education in Vöcklabruck and Wels.

2012–2018 Studies of Mathematics in the joint program of University of Graz and Graz University of Technology.

15.3.2018 master degree (Dipl.-Ing) with distinction.

2) Employment

since 1.4.2018 PhD position at Institut für Diskrete Mathematik, funded until 29.2.2020 by TU Graz - Hochschulraum-Strukturmittel. PhD advisor W. Woess.

3) Teaching

Excercise courses in Mathematics I, II for Mechanical Engineering, Discrete Mathematics for Telematics, and Linear Algebra for Mathematics.