

STAND-ALONE PROJECT - FINAL REPORT

P19115-N18 Project number

Project title

Random walks, random configurations, and horocyclic products

Project leader

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Project website

<http://www.math.tugraz.at/~woess/#fwf>

Table of Contents

Part 1. Summary for public relations work (English & German) page 2

The project's most important results (scientific advances) from the project leader's point of view are presented in the Summary on a single page in a way that is accessible to the general public. The summary is submitted as hardcopy and also electronically both in German and in English. The summaries will be made available via the FWF's project database. The FWF will not edit the summaries, so the authors bear full responsibility for the contents.

Parts 2-4 are addressed to reviewers.

Part 2. Brief project report page 4

2.1. Report on the scientific work page 4

2.2. Personnel development after termination of the project page 8

2.3. Effects of the project – statement on the overall impact page 8

Part 3. Information on project participants..... page 9

3.1. Project personnel and employment periods within the project..... page 9

3.2. Further personnel of the research group in the project period page 9

Part 4. Attachments..... page 11

4.1. Scientific publications page 11

4.2. Project related participation in international conferences page 13

4.3. Development of collaborations page 16

4.4. PhD theses / diploma theses..... page 16

4.5. Effects of the project regarding the scientific field..... page 17

4.6. Applications for follow-up projects..... page 18

Part 5. Zusammenarbeit mit dem FWFpage 19

Part 5 provides an opportunity to report to the FWF on interactions with the administration during the course of the project.

1. Summary for public relations work

English. A random walk is a random process on a graph (network), where a particle (walker) moves randomly from point to point. Randomness is provided by a transition matrix which encodes the probabilities to move to a next point, given the actual position. These probabilities are assumed to be adapted to the underlying graph structure by some conditions that have to be specified from case to case. The theme of the project has been twofold: on one hand, we considered random configurations of an increasing of fluctuating number of particles that occupy points in the graph. One such model is internal diffusion limited aggregation (IDLA), where one source point emit successive particles, each of which performs a random walk until it occupies the first site that has so far remained unoccupied. The question here is to understand the asymptotic shape of the set of occupied sites, as the number of particles increases. Another model is branching random walk, where the particles, while performing random motions, produce offspring which also move at random. A third model is given by lamplighter random walks, where a “lamplighter” performs a random walk on a graph (road network) and randomly changes the states of “lamps” situated at the points that (s)he successively visits. This last model leads to the other main topic of the project. Namely, the state space of a lamplighter walk on the integer line can be understood as the horocyclic product of two regular trees, which in the past has lead to a very good understanding of such walks. Thus, we have also studied random processes, harmonic functions and related issues on more general types of horocyclic products.

Among the achievements of the project, one of the project collaborators has produced a PhD thesis on three internal aggregation models, one of which is IDLA. This is well understood on integer grids, but the main body of this work concerns non-homogeneous structures, in particular the “comb lattice”, which has required new ideas in order to describe rigorously the diamond-like asymptotic shape of the clusters.

Other highlights concern random walks, spectral theory and spatial asymptotics on so-called affine buildings and their horocyclic products. Also, a surprising “discovery” was the relation between the spectra of the transition operators lamplighter random walks and subcriticality of percolation on the underlying graph. Site percolation is a random model where the points of a graph are “open” with probability p and “closed “ with probability $1-p$, independently of each other, with a probability parameter p . We proved that for the basic lamplighter walk, the spectrum is pure point and can be understood completely, when percolation with $p=1/L$ is such that all clusters of open points are finite. Here, L is the number of possible different states (eg colours or intensity) of the lamps at the different sites.

Deutsch. Eine Irrfahrt (random walk) ist ein Zufallsprozess auf einem Graphen (Netzwerk), wo sich ein Partikel (walker) zufällig von Punkt zu Punkt bewegt. Der Zufall wird hierbei durch eine Übergangsmatrix beschrieben. Bei gegebener aktueller Position gibt diese die Wahrscheinlichkeiten an, mit denen sich das Partikel im nächsten Schritt zu einem der anderen Punkte bewegt. Es wird vorausgesetzt, dass diese Übergangswahrscheinlichkeiten an die zugrunde liegende Graphenstruktur durch fallweise zu spezifizierende Bedingungen angepasst sind. Das Projekt hatte zwei Hauptthemen. Einerseits ging es um die Untersuchung von Zufallskonfigurationen einer zunehmenden oder fluktuierenden Anzahl von Partikeln in den Punkten des Graphen. Ein solches Modell ist „internal diffusion limited aggregation“ (IDLA), wo ein Quellpunkt sukzessiv Partikel ausschickt, deren jeder eine Zufallswanderung vollführt, bis er sich am ersten Punkt festsetzt, der bislang unbesetzt war. Die Frage ist hier die asymptotische Form der Menge der besetzten Punkte, wenn die Anzahl der Partikel zu-nimmt. Ein weiteres Modell ist die verzweigende Irrfahrt, wo die Partikel, während sie eine Zufallsbewegung ausführen, Nachfahren erzeugen, die sich wiederum zufällig fortbewegen. Ein drittes Modell ist durch sogenannte „lamplighter“-Irrfahrten gegeben, wo eine Person eine Irrfahrt in einem Graphen (Strassennetz) vollführt und die Zustände von Lampen, die sich an den sukzessiv besuchten Knoten des Netzes befinden, zufällig verändert. Dieses letzte Modell führt zum zweiten Hauptthema des Projekts. Es kann nämlich der Zustands-raum einer „lamplighter“-Irrfahrt auf dem ganzzahligen Liniengraphen als das horozyklische Produkt zweier regulärer Bäume verstanden werden. Im Vorläuferprojekt hat dies zu einem sehr guten Verständnis solcher Irrfahrten geführt. Daher haben wir nun auch Zufallsprozesse, harmonische Funktionen und verwandte Themen auf allgemeineren Typen von horozyklischen Produkten studiert. Unter den Ergebnissen des Projekts hat einer der Mitarbeiter eine Dissertation über drei innere Aggregationsmodelle erarbeitet, darunter IDLA. Hierüber ist schon viel bekannt für die Zahlengitter in beliebiger Dimension, aber der Hauptteil dieser Arbeit betrifft nicht-homogene Strukturen, insbesondere das „Kamm-Gitter“, welches neue Ideen erforderte, um die Diamant-förmige asymptotische Gestalt der cluster rigoros zu beschreiben. Andere Projektergebnisse betreffen Irrfahrten, Spektraltheorie und räumliche Asymptotik auf sogenannten „affine buildings“ und ihren horozyklischen Produkten. Eine weitere, überraschende „Entdeckung“ war der Zusammenhang zwischen den Spektren von Übergangsoperatoren von „lamplighter“-Irrfahrten und der Perkolation auf dem Basisgraphen. Knotenperkolation ist ein Zufallsmodell wo die Punkte eines Graphen mit Wahrscheinlichkeit p „offen“ und mit Wahrscheinlichkeit $1-p$ „geschlossen“ sind, voneinander unabhängig und mit Wahrscheinlichkeitsparameter p . Wir konnten zeigen, dass für die typische „lamplighter“-Irrfahrt ein reines Punktspektrum vorliegt und komplett verstanden wird, sofern die Perkolation mit $p=1/L$ so ist, das alle Komponenten offener Punkte endlich sind. Hier ist L die Anzahl der verschiedenen möglichen Zustände (zB Farben oder Intensität) der Lampen an den Punkten.

2. Brief project report

2.1 Report on the scientific work

2.1.1 Information on the development of the research work (2000 characters excl. spaces) - Overall scientific concept and goals

The project proposal is still available under <http://www.math.tugraz.at/~woess/Proposal.pdf>.

A central theme can be subsumed as “*random configurations driven by random walks*”. On one hand, this refers to the stochastic model of internal diffusion limited aggregation (IDLA) and related aggregation models, and on the other hand, it concerns the configurations arising in lamplighter random walks. In the case of lamplighters over the integers, the state space of the process can be described geometrically as the horocyclic product of two regular trees. In this sense, the study of more general types of horocyclic products is a related topic, and the study of the structure of such products as well as random walks, spectral theory, and related issues has also been a theme of this project.

Additional related topics were incorporated in relation with the interests and interactions of the involved personnel. As already stated in a previous report, in the employment of project personnel it has been my explicit strategy to give preference to the best candidates and their competences, possibly at the cost of making slight deviations from the initially proposed tracks and to include additional aspects. This applies, in particular, to the work of Dr. Agelos Georgakopoulos, whose multiple interests have enriched the project substantially towards more graph-theoretical directions.

2.1.2 most important results and brief description of their significance (main points)

2.1.3 information on the running of the project, use of the available funding and where appropriate any changes relating to the original project plan

- *The development of the project was such that these two items are hard to separate & are presented jointly.*
- *Regarding project personnel and employment periods, see Section 3.*
- *Reference numbers refer to the publication list in Section 4.1. A few selected publications are highlighted in boldface.*

The project started on October 1, 2006 and ended on March 31, 2011. The project funding was for one PhD and one PostDoc for 3 years each. It was officially interrupted from December 16, 2008 until April 15, 2009, but the project research itself was not interrupted at all. This was due to the following facts. (1) The PhD student employed in the project, Wilfried HUSS, had the opportunity to replace two members of the institute during periods of leave. On those two occasions, he held an assistant position. (2) The first PostDoc employed in the project, James PARKINSON, left earlier than originally planned because he obtained a tenure track position at Sydney University. The project related collaboration with him has not ceased after his departure. For the project, these two occurrences had the effect that some of the funding could be used for other young scientists.

Wilfried HUSS, the PhD student employed within this project, worked on internal aggregation models on non-homogeneous graphs, with initial focus on internal diffusion limited aggregation (IDLA). His first paper [4] adapts a result of BLACHERE and BROFFERIO (obtained within my previous FWF project P15577-N05) from IDLA on groups with exponential growth to IDLA on non-amenable graphs. Huss' subsequent work focussed on

IDLA on the *comb lattice*. This is the spanning tree of the 2-dimensional grid, where all horizontal edges are removed except those on the x -axis. The study of IDLA on this graph is harder than on the grid itself. In the same period, a strong group around Yuval Peres has developed new methods for dealing with internal aggregation on grids, and Huss has been influenced by that work. In particular, IDLA is linked with two other models, namely the *divisible sandpile* and *rotor router aggregation*. These two are deterministic, but all experimental data exhibit striking similarity with IDLA. Especially, the divisible sandpile is a good base before exploring the other two. In his thesis, Huss has determined the asymptotic shape of the divisible sandpile configuration started with n particles at the origin of the comb, as n grows. Subsequently, in part in collaboration with E. SAVA, he has determined the asymptotic shape of rotor router aggregation on the comb exactly for a specific natural initial rotor configuration, and with reasonable inner and outer bounds in general. Finally, they have found a good inner estimate for the asymptotic shape of IDLA on the comb. These results are presented in Huss' thesis [9]. Besides the note [4], the papers [22], [23] have been extracted from the thesis, resp. report on subsequent research, and were submitted to international journals.

There is a "side result" published in a note by Huss, Sava and myself [10]. It does not concern exactly one of the proposal's topics, but there is a strong interplay with the general research theme. Questions regarding growth of groups, graphs, or languages are closely linked with random walks. Here, we use Markov chain methods to prove an "infinite" analogue of a classical result regarding the entropy of sofic systems, resp. growth of regular languages.

Groups and languages are also present in my papers [17] and [15]. [15] contains a complete description of the asymptotic behaviour of n -step transition probabilities of finite range random walks on virtually free groups. For free groups, this has been obtained in an impressive paper of Lalley [Ann. Probab. 21 (1993), 2084-2130], who suggested that the result for virtually free groups would follow from his later paper on regular languages [Contemporary Math. 287 (2001), 201-230]. However, it is not clear how the conditions of that paper can be implemented for general virtually free groups, and [15] gives a complete answer via a different approach to that problem. This answers a question of Saloff-Coste.

According to the original proposal, the designated PostDoc was James PARKINSON. The reason why I was able to get such a strong group member was that I knew about his ongoing PhD work before it was finished, and contacted him quite a while before that. When his very impressive thesis became known, he received invitations from the US (University of Wisconsin-Madison and Cornell University). For this reason he postponed his stay in Graz for a year.

In 2006 I met Sebastian MÜLLER who was a PhD student of Nina Gantert at Münster. His work on branching random walks fitted perfectly into the theme "Random configurations driven by random walks". In view of the delayed arrival of Parkinson, I offered Müller to join my group after his PhD (of which I became the 2nd referee) and to work in the FWF project for some time, while applying for a some other long term grant. After being funded by the project for 5 months in 2007, he received a grant of Deutsche Forschungsgemeinschaft for staying in Graz for 2 more years. His contributions are [1], [2] and (only in part officially associated with the present project) [12], but also his later work with other members of my group became possible only through the initial involvement in FWF P19115. [1] and [2] explore the transient, resp. recurrent regimes of branching random walks in relation with the underlying branching rules. My favorite among those three is [12], where together with Gilch he provides very nice extensions of my previous work with Naghibeda on random walks on trees, this time not necessarily with finitely many cone types, and furthermore also including results on the asymptotic entropy.

James PARKINSON's papers [5] and [16] are mostly based on research prior to his arrival in autumn 2007. [16] is closely related to our subsequent work here in Graz. The two papers [26] and [27] are located at the center of the topic of the project. In [26], we study the horocyclic product of two affine buildings with the same "dimensions". It turns out that this is related with certain twin buildings, and with specific groups appearing in that context. Besides this structural aspect, we are studying the spectrum of the simple random walk operator of the 1-skeleton of that product. It is expected to be pure point, but work is still in progress because of the appearance of a problem that we still have to overcome. The whole spectrum is related with the spectrum of the classical Laplace operator of a rhombus made up by two equilateral triangles, with Dirichlet condition at the boundary. One could say that we have determined "half" of the spectrum, which leads to a point spectrum part of the horocyclic product. It corresponds to functions on the rhombus which are even (invariant under reflection along the diagonal), while the odd half needs additional work that is still in progress. In [27], we are studying aspects of the metric geometry of buildings, namely regularity of sequences in the spirit of Kaimanovich ["Lyapunov exponents, symmetric spaces and a multiplicative ergodic theorem for semisimple Lie groups"; translation in J. Soviet Math. 47 (1989), 2387–2398]. This is of importance for random walks on buildings as well as their horocyclic products, since it will allow to determine the rate of escape, a central limit theorem, convergence to the boundary and related issues. The paper [13] also studies random walks on buildings and was written by Parkinson with a visitor, Bruno Schapira from Paris. Finally, [28] was begun during Parkinson's stay in Graz.

On the occasion of a visit at Reinhard Diestel's research group in Hamburg, I met his PhD student Agelos GEORGAKOPOULOS, an excellent graph theorist with unusually wide interests. He had been working on topological boundaries of infinite graphs and has produced impressive papers. He joined the project in Graz for its last 2 years, as Parkinson's successor. Georgakopoulos is interested in Gromov-hyperbolic graphs, in harmonic functions with finite power (energy), in planar Cayley graphs, and in Brownian motion on metric graphs whose edge lengths are summable. As mentioned above, his presence has induced a partial shift towards more graph theoretical issues. A good part of them fit very well into the project theme. A first open problem that I suggested to him, and that Anders Karlsson and myself had tried to solve a few years earlier, was to decide whether the lamplighter graph over a tree can carry non-constant Dirichlet harmonic functions. In fact, this was one of the questions addressed in the project proposal. It was amazing to see how quickly Agelos solved this problem completely for general lamplighter graphs [7]. Related questions regarding electric currents and harmonic functions with finite power on graphs with finite total sum of edge resistances are the subject of his paper [8]. Recently, he has started a promising collaboration with Konrad KOLESKO on Brownian motion on metric graphs with the latter property (their first joint paper is in preparation). Already papers [7] and [8] could be considered completely satisfactory to justify Georgakopoulos' collaboration in the project, so that I was certainly glad that he also continued his research in "pure" graph theory, even if not always directly linked with the original project proposal. Among that work, I would like to point out, in particular, his complete classification of planar cubic Cayley graphs [18], [19], [20].

In autumn 2010, a period when Wilfried Huss' salary came from other sources, I decided to use the available FWF personnel funding to bring two young PhD students to the research group for relatively short periods of 3 months each. This led to an additional stimulus for the whole research group.

Mathias HAMANN is an excellent PhD student of Reinhard Diestel, who has already written several papers on infinite graphs, in particular hyperbolic graphs, group actions, etc. He collaborated with Georgakopoulos and contributed the paper [30] to the outcomes of P19115-N18. Needless to say that such structural investigations regarding hyperbolic graphs are important for all types of research in my group, because the interplay between structural features and probabilistic, analytic and other questions is *the* basic theme of our research.

The other young PhD student who joined the team for those 3 months was Konrad KOLESKO from Wroclaw (Poland), where he works in the group of Damek and Buracewski. His main interest is stochastic affine recursions and the ramifications of that theory. This is linked with my own work on random walks on groups that are in some sense related with the affine group (in particular, horocyclic products), as well as with my recent cooperation with Marc Peigné (Tours, France) on stochastic dynamical systems [37], [38]. Kolesko contributed the paper [24] to the outcomes of the project. As mentioned above, he also started a promising cooperation with Georgakopoulos on Brownian motion.

Practically all research questions addressed in the original project proposal have been subjects of the work in my research group in the last years, but not all this was carried out just by the project employees.

For example, the Poisson boundary of lamplighter random walks on trees was first described by KARLSSON and myself [Geometriae Dedicata 124 (2007) 95-107], acknowledging partial support of the previous FWF project, and this has been taken up by my PhD student Ecaterina SAVA [33].

The study of the spectrum of lamplighter transition operators is one of the project's highlights: this is the paper [3], joint with LEHNER and with one of the PostDocs of the previous FWF project, NEUHAUSER. There, we show that for a symmetric “switch-walk-switch” lamplighter walk on a wreath product of an infinite group G with a finite group H , a sufficient condition for having pure point spectrum is that site percolation on the underlying Cayley graph of G with probability parameter $p=1/|H|$ has finite clusters. This has later been generalised by LEHNER [32] and is used by LEHNER and WAGNER [39] to disprove a conjecture of Atiyah.

Another project related highlight is [11], in collaboration with BENDIKOV, SALOFF-COSTE (one of the partners indicated in the project proposal) and SALVATORI. It is based on a question motivated by [29], which is still being written: the horocyclic product on a tree and hyperbolic plane is a Riemannian complex which appears, for example, as a space on which the Baumslag-Solitar group acts with compact quotient. In view of its singularities, how can one define rigorously Brownian motion on that space? General answers in a more general context are given in [11], a basis for the follow-up project.

Parallel to [29], BROFFERIO, SALVATORI and myself have concluded the paper [25] on Brownian motion on $Sol(p,q)$, the horocyclic product of two hyperbolic planes with possibly different curvatures.

Some final words are due on the **external project collaborators** and their interaction with the project staff. Five names were included in the proposal. Of these, L. SALOFF-COSTE did not manage to come to Graz (he is now chairman of the Department of Mathematics at Cornell), but the collaboration with him has not ceased.

Sara BROFFERIO has visited Graz several times, in 2007 (one week), 2008 (two weeks) and 2009 (the Boundaries workshop). She interacted mainly with Wilfried HUSS (discussing IDLA) who visited her in Paris-Orsay in October 2006, and with myself plus Maura SALVATORI, who also visited Graz. An outcome of these visits is [26].

Laurent BARTHOLDI came to Graz for a month in spring 2008; see the collaboration with PARKINSON and myself [23].

Donald CARTWRIGHT came to Graz for a month in early 2009, and again to the Boundaries workshop 6 months later. He is a long time collaboration partner.

Vadim KAIMANOVICH is also a long time collaboration partner. He came to Graz for a week in November, 2008, and again for the Boundaries workshop in 2009. He started a very promising research with GEORGAKOPOULOS, who visited him in Bremen for a month in late 2009; a joint paper is in preparation.

As previously, a good part of these visits did not have to be paid by project money, because I managed to find other funds for those purposes. In this way I could save money for the salaries of project employees. Apart from those salaries, the FWF funding was used for conference participations and some research stays of the project personnel, while I avoided to use project money for my own conference participations.

There were various further short and longer term visitors from whom we benefitted; a quite complete list can be found at <http://www.math.tugraz.at/mathc/new/index.php?link=guests>.

2.2. Personnel development after termination of the project

– importance of the project for the scientific careers of those involved

- Wilfried HUSS : PostDoc position at University of Siegen, Germany, in the research group of Uta FREIBERG.
- Sebastian MÜLLER: Marie Curie grant and subsequently maitre de conference position (tenure track), both at the University of Aix-Marseille I, France.
- James PARKINSON: Lecturer (tenure track) at the University of Sydney, Australia.
- Agelos GEORGAKOPOULOS: Visiting position at the University of Ottawa, Canada, in the research group of Vadim A. KAIMANOVICH, and subsequently (starting in autumn, 2011) at Université de Genève in the research group of Tatiana SMIRNOVA-NAGNIBEDA.
- Mathias HAMANN: PhD position at the University of Hamburg in the research group of Reinhard DIESTEL.
- Konrad KOLESKO: PhD position at the University of Wrocław in the research group of Ewa DAMEK and Dariusz BURACEWSKI.

As in the preceding project P15577-N05, one can see that the project collaborators have spread out over the world and are successfully continuing their research career.

2.3. Effects of the project - statement on the overall impact of this project

When I came to TU Graz in 1999, the small department of which I became the head was equipped with 2 assistant positions, both held by tenured persons working in quite different areas than myself. In the meantime there are three regular university positions besides myself (held by Franz Lehner, Lorenz Gilch and Christoph Temmel, see Section 4). Since 2002, I have run two subsequent FWF projects, each one with the funding corresponding to one PostDoc and one PhD position. These two projects have been the backbone for building up a vibrant research group of 9-12 people on the whole. The FWF projects have really been essential for this. Let me exemplify my way of operating. James Parkinson joined my group within FWF project P19115-N18 somewhat later and left for a tenured position in Sydney earlier than originally planned. He was not "lost" for the research group, because the cooperation continued (I visited Sydney in summer 2009, Parkinson visited Graz in autumn 2009, and two of my master students have worked on their master theses during a semester at Sydney Univ.). At the same time, I could use the project funding not only to employ another very good PostDoc, Agelos Georgakopoulos, but also to attract other young people

to come to Graz. The strategy has been as follows: “join my group and apply for funding; if your own application is not successful I’ll be glad to employ you within the FWF project”. With a success rate of more than 50% for those “other applications”, this means that I could enlarge my group by twice as many persons as provided directly by the FWF funding. But I could not have done this without the FWF project in the back. For example, Elisabetta Candellero from Torino had first come to Graz as an Erasmus exchange student and wrote her master thesis under my supervision. Then I was able to find some interim funding for a year for her to start working toward PhD, and suggested to her to apply for a grant from the Austrian Academy of Sciences. In case this might have failed, I was able to promise her that she could join the FWF project, while without the latter I could not even have tried to get her to Graz. At the end, she did obtain that Academy grant, so that the FWF funding which had been initially reserved for Elisabetta could be used for other young mathematicians to work in the research group. A few years earlier, a similar approach worked for Ecaterina Sava from Romania and for Sebastian Müller from Germany. In this way, I have been able to build up a nice group of young persons working on related topics, but not monothematically.

With an open mind of bringing young people to Graz, I have been able to offer research opportunities to more young persons than what is just provided by the FWF funding for one PhD and one PostDoc.

So I can say that this as well as the preceding FWF project have had an extremely positive impact on building up my research group, quite far beyond the formal scope of the project. For this reason, in this report, it has not been reasonable to completely separate the formal project employees from the other members of the group, because all of them profited from and contributed to the project(s) directly or indirectly.

3. Information on project participants

Official project duration (all dates are in the format day/month/year):
1/10/2006 – 15/12/2008 and 15/4/2009 – 31/3/2011.

3.1. Project personnel and employment periods within the project

PhD student Wilfried HUSS: 1/10/2006 – 15/9/2008 and 1/8/2009 – 30/9/2009.
In the periods 16/9/2008 – 31/7/2009 and 1/10/2009 – 31/12/2010, he held PhD positions (univ. assistant) funded by TU Graz.

PostDoc Dr. Sebastian MÜLLER: 1/5/2007 – 30/9/2007
In the period 1/10/2007 – 30/9/2009 he remained within the research group, funded by a grant of Deutsche Forschungsgemeinschaft (DFG).

PostDoc Dr. James PARKINSON: 1/11/2007 – 15/12/2008

PostDoc Dr. Agelos GEORGAKOPOULOS: 15/4/2009 – 31/3/2011

PhD student Mathias HAMANN: 16/9/2010 – 15/12/2010

PhD student Konrad KOLESKO: 16/9/2010 – 15/12/2010

3.2. Further personnel of the research group in the project period (funded by other sources)

[These persons are listed here because of their strong interaction with the project and also included in the table below]:

Senior university assistant Dozent Dr. Franz LEHNER (regular position at TU Graz, now tenured, 1/9/2010 – 28/2/2011 on leave 1/2/2009 – 31/7/2009 at Univ. Geneva)

Dr. Lorenz GILCH: until 31/8/2008 PhD position (univ. assistant) funded by TU Graz, 1/9/2009 – 28/2/2010 DFG grant at TU Graz, since 1/3/2010 senior assistant position at TU Graz (1/9/2010 – 28/2/2011 on leave with DFG grant at Univ. Geneva)

PhD student Christoph TEMMEL: since 1/10/2008 PhD position (univ. assistant) funded by TU Graz (1/10/2009 – 30/6/2010 on leave at Univ. Marseille).

Dr. Ecaterina SAVA: 1/10/2006 – 31/12/2010 PhD position (within the NAWI Graz cooperation project of the two Graz universities) funded by TU Graz. Since 1/1/2011 coordinator of the DK (graduate program) “Discrete Mathematics” funded by FWF at TU Graz, Univ. Graz and Univ. Leoben.

PhD Student Elisabetta CANDELLERO: 1/4/2009 – 31/3/2010 PhD position (within the NAWI Graz cooperation project of the two Graz universities) funded by TU Graz. Since 1/4/2010 PhD grant of the Austrian Academy of Sciences.

PhD Student Tetiana BOIKO: since 1/10/2010 PhD position (within the NAWI Graz cooperation project of the two Graz universities) funded by TU Graz.

Dr. Florian SOBIECZKY: 1/10/2006 – 31/3/2009, PostDoc within his own FWF project P18703. Then at Jena University, but commuting between Graz and Jena, currently at University of Colorado – Boulder, still linked with the group.

not funded by this FWF project			funded by the FWF (this project)		
co-workers	number	Person-months	co-workers	number	Person-months
non-scientific co-workers			non-scientific co-workers		
diploma students	3	6 (30)	diploma students		
PhD students	6	28 (138)	PhD students	3	31,5
post-doctoral	4	14 (69,5)	post-doctoral	3	42
co-workers			co-workers		
co-workers with “Habilitation” (professorial qualifications)	1	10 (50)	co-workers with “Habilitation” (professorial qualifications)		
professors	1	10 (50)	professors		

Comment on the above table: While the figures in its right half are obvious, the correct quantification in the left half is difficult. The permanent interaction within the whole group is such that each member has in some way contributed to the project. Regarding the number of person-months in the left half, I have computed a “flat rate” of 20% of the actual number of months for the listed employees. In some cases (such as the diploma students), this is too much, in other cases (such as myself, or the employment of Huss outside of the project) it is too little. The numbers of co-workers are of course absolute numbers. Here, one has to take into account that Huss and Müller figure on both sides, because they were employed within the project as well as outside in different periods.

4. Attachments

List 4.1. scientific publications

All project publications that have appeared, resp. have been accepted for publication are peer-reviewed. Papers that are currently submitted as well as those which are still being brought to paper are / will also be subject to a peer-reviewing procedure. Some of the publications (in a transitory phase of project staff, or in the case of external co-authors) are also attributed to other projects, resp. funding. All papers are accessible on ArXiv. All papers of which I am a (co-)author are freely accessible on my website at <http://www.math.tu-graz.ac.at/~woess/#papers>.

4.1.1. Project related publications that have already appeared in print

(FWF support is acknowledged in all publications. In some of them, the support is partial. By mistake, in a few of them the wrong project number – of Sobieczky's project – was indicated.)

[1] S. Müller: Recurrence for branching Markov chains. *Electronic Communications in Probability* 13 (2008), 576–605.

[2] S. Müller: A criterion for transience for multidimensional BRWRE. *Electronic Journal of Probability* 13 (2008), 1189–1202.

[3] F. Lehner, M. Neuhauser and W. Woess: On the spectrum of lamplighter groups and percolation clusters. *Mathematische Annalen* 342 (2008) 69-89.
DOI: 10.1007/s00208-008-0222-7

[4] W. Huss: Internal diffusion-limited aggregation on non-amenable graphs. *Electronic Communications in Probability* 13 (2008), 272–279.

[5] J. Parkinson, A. Ram and C. Schwer: Combinatorics in affine flag varieties. *Journal of Algebra* 321 (2009), 3469–3493. DOI:10.1016/j.jalgebra.2008.04.015

[6] M. DeVos, A. Georgakopoulos, B. Mohar, and R. Samal: An Eberhard-like theorem for pentagons and heptagons. *Discrete & Computational Geometry*, 44, (2010) 931–945.
DOI: 10.1007/s00454-010-9264-1

[7] A. Georgakopoulos: Lamplighter graphs do not admit harmonic functions of finite energy. *Proc. Amer. Math. Soc.* 138 (2010), 3057–3061.
DOI: 10.1090/S0002-9939-2010-10279-4

[8] A. Georgakopoulos: Uniqueness of electrical currents in a network of finite total resistance. *J. London Math. Soc.* 82 (2010), 256–272. DOI: 10.1112/jlms/jdq034

[9] W. Huss: *Internal Aggregation Models*. PhD Thesis, TU Graz, 2010.

[10] W. Huss, E. Sava and W. Woess: Entropy sensitivity of languages defined by infinite automata, via Markov chains with forbidden transitions. *Theoretical Computer Science* 411 (2010), 3917–3922. DOI:10.1016/j.tcs.2010.07.020

[11] A. Bendikov, L. Saloff-Coste, M. Salvatori and W. Woess: The heat semigroup and Brownian motion on strip complexes. *Advances in Mathematics* 226 (2011), 992-1055.
DOI:10.1016/j.aim.2010.07.014

[12] L. Gilch and S. Müller: **Random Walks on Directed Covers of Graphs.** *Journal of Theoretical Probability* 24 (2011), 118-149. DOI: 10.1007/s10959-009-0256-0

[13] B. Schapira and J. Parkinson: A local limit theorem for random walks on the chambers of A_2 buildings. In: Random Walks, Boundaries, and Spectra (D. Lenz, F. Sobieczky and W. Woess, eds.), pp. 15-53. *Progress in Probability*, Birkhäuser (2011).

[14] A. Georgakopoulos: Graph topologies induced by edge lengths. *Discrete Mathematics* 311, (2011), 1523–1542. [DOI:10.1016/j.disc.2011.02.012](https://doi.org/10.1016/j.disc.2011.02.012)

4.1.2. Project publications that have been accepted for print

[15] W. Woess: Context-free pairs of groups. II - Cuts, tree sets, and random walks. *Discrete Mathematics*, in print.

4.1.3. Project publications that have been submitted

[16] A. Ram and J. Parkinson: Alcove walks, buildings, symmetric functions and representations.

[17] T. Ceccherini-Silberstein and W. Woess: Context-free pairs of groups. I – Context-free pairs and graphs.

[18] A. Georgakopoulos: A group has a planar Cayley complex if and only if it has a VAP-free Cayley graph.

[19] A. Georgakopoulos: The planar cubic Cayley graphs of connectivity 2.

[20] A. Georgakopoulos: The planar cubic Cayley graphs.

[21] A. Georgakopoulos: Cycle decompositions: from graphs to continua.

[22] W. Huss and E. Sava: Internal aggregation models on the comb lattice.

[23] W. Huss and E. Sava: Rotor router aggregation on the comb.

[24] K. Kolesko: Tail homogeneity of invariant measure of multidimensional stochastic recursions in a critical case.

[25] S. Brofferio, M. Salvatori and W. Woess: Brownian motion and harmonic functions on $\text{Sol}(p,q)$.

4.1.4. Project publications in phase of preparation

[26] L. Bartholdi, J. Parkinson and W. Woess: Horocyclic products of affine buildings.

[27] J. Parkinson and W. Woess: Regular sequences in affine buildings.

[28] J. Parkinson: Representations and the Plancherel theorem for rank 2 affine Hecke algebras.

[29] A. Bendikov, L. Saloff-Coste, M. Salvatori and W. Woess: Brownian motion and harmonic functions on treebolic space.

[30] Matthias Hamann: On the tree-likeness of hyperbolic spaces.

4.1.5. Publications having close relation with the project

In the period of the FWF project P19115-N18, the research group at Institut für Mathematische Strukturtheorie has produced a number of further publication that did not directly rely on the FWF funding, but are – as regards to contents and involved persons – closely related with the project, so that the outcomes cannot be seen as completely separated from the project work (example: [32] and [39] are continuations of and conditioned by [3]). Compare with the Statement on the overall impact of this project in the main part of this report. These publications are listed here.

[31] L. Saloff-Coste and W. Woess: Computations of spectral radii on G-spaces. *Contemporary Math.* 484 (2009) 195–218.

[32] F. Lehner: On the eigenspaces of lamplighter random walks and percolation clusters on graphs. *Proc. Amer. Math. Soc.* 137 (2009), 2631– 2637.
DOI:10.1090/S0002-9939-09-09869-4

[33] E. Sava: A note on the Poisson boundary of lamplighter random walks. *Monatshefte für Mathematik* 159 (2010), 379–396. DOI: 10.1007/s00605-009-0103-5

[34] L. Gilch: Asymptotic Entropy of Random Walks on Free Products. *Electronic Journal of Probability* 16 (2011), 76 –105.

[35] D. Lenz, F. Sobieczky and W. Woess: Random Walks, Boundaries, and Spectra. Proceedings of two workshops, Graz and St. Kathrein 2009. Progress in Probability, vol 64. Birkhäuser, Basel, 2011. ISBN 978-3-0346-0243-3

[36] E. Candellero and L. Gilch: Phase Transitions for Random Walk Asymptotics on Free Products of Groups. *Random Structures and Algorithms*, in print.

[37+38] M. Peigné and W. Woess: Stochastic dynamical systems with weak contractivity properties, parts I and II. *Colloquium Math.*, in print.

[39] F. Lehner and Stephan Wagner: Free Lamplighter Groups and a Question of Atiyah. *American J. Math.*, in print.

[40] E. Candellero, L. Gilch and S. Müller: Branching Random Walks on Free Products of Groups. *J. London Math. Soc.*, in print.

List 4.2. project-related participation in international scientific conferences

The conference participations marked with (*) were funded by the project. For the other conference participations, different funding was found.

4.2.1. Conference participations - invited lectures

1) W. Woess: "Horocyclic products of trees", Conference on "Random Walks on Groups", CIRM, Marseille, France, 5-9 February 2007.

2) W. Woess: "Non-backtracking random walks and cogrowth of graphs", Workshop on "Amenability Beyond Groups", ESI, Vienna, Austria, Feb-March 2007.

3) W. Woess: "Lamplighter random walks", conference "Non-classical Interacting Random Walks", Mathematisches Forschungsinstitut Oberwolfach, Germany, 20-25 May 2007.

4) W. Woess: *“The Poisson boundary of lamplighter random walks on the homogeneous tree”*, conference “Recent Developments in Random Walks”, Durham University, UK, 2-7 July 2007.

5) W. Woess: *“Horocyclic products of trees and of hyperbolic spaces”*, Workshop on “Algebraic, Geometric and Probabilistic Aspects of Amenability”, ESI, Vienna, Austria, June-July 2007.

6) W. Woess: *“Horocyclic products of trees and of hyperbolic spaces”*, International Conference “Spectral Analysis in Geometry and Number Theory”, Nagoya University, Japan, 6-10 August 2007.

7) W. Woess: *“Random configurations driven by random walks”*, Conference on “Random, Growing, Infinite Networks”, Heinrich Fabri Institut, Blaubeuren, Germany, 9-12 January 2008.

8) J. Parkinson: *“Horocyclic products of affine buildings”*, conference “Harmonic Analysis and Group Actions in Analysis, Geometry and Probability”, Zakopane, Poland, 12-20 April 2008. (*)

9) W. Woess: *“Horocyclic products of trees and of hyperbolic spaces”*, conference “Harmonic Analysis and Group Actions in Analysis, Geometry and Probability”, Zakopane, Poland, 12-20 April 2008.

10) J. Parkinson: *Horocyclic products of affine buildings*, meeting on “Groups and Infinite Graphs”, ESI, Vienna, 25-27 August 2008.

11) J. Parkinson: *“The horocyclic product of affine buildings”*, conference “Buildings 2008”, Münster, Germany, 29 Sept – 1 October 2008. (*)

12) W. Woess: *“The spectrum of lamplighter groups and percolation clusters”*, Workshop on Structural Probability, ESI, Vienna, Austria, 3-7 November 2008.

13) W. Woess: *“Brownian motion and harmonic functions on treebolic space”*, Symposium in Honor of Vadim Kaimanovich, Jacobs University, Bremen, Germany, 28 November 2009.

14) A. Georgakopoulos: *“Hyperbolic graphs, fractal boundaries, and graph limits”*, Workshop “Graph Theory”, Mathematisches Forschungsinstitut Oberwolfach, Germany, 22-26 February 2010. (*)

15) W. Woess: *“Weakly contractive stochastic dynamical systems”*, Conference “Analysis, Geometry and Probability Related to Group Actions”, Zakopane, Poland, 17-24 April 2010. (*)

4.2.2. Conference participations – lectures

1) W. Huss: Special session *“Internal diffusion limited aggregation on graphs and groups”* within the conference “Random Walks on Groups”, Marseille, France, 5-9 February 2007. (*)

2) S. Müller: *“Criteria for transience of branching Markov chains”*, 15th European Young Statistician Meeting, Bilbao, Spain, 10-14 September 2007. (*)

3) A. Georgakopoulos: *“The Dirichlet problem in a network of finite total resistance”*, Conference on Algebraic Graph Theory 2009, InterUniversity Centre Dubrovnik, Croatia, 1-7 June 2009. (*)

- 4) A. Georgakopoulos: *“Uniqueness of currents in a network of finite total resistance”*, Workshop on “Boundaries”, TU Graz, 29 June – 4 July 2009.
- 5) A. Georgakopoulos: *“Every rayless graph has an unfriendly partition”*, EuroComb09, Bordeaux, France, 7-11 September 2009. (*)
- 6) W. Woess: *“Context-free pairs of groups”*, lecture, Conference on Algebraic Graph Theory 2009, InterUniversity Centre Dubrovnik, Croatia, 1-7 June 2009.
- 7) W. Woess: *“Brownian motion and harmonic functions on treebolic space”*, lecture, ÖMG – DMV Congress, TU Graz, Austria, 21-25 September 2009.

4.2.3. Conference participations – posters

- 1) W. Huss: poster presentation at the conference “Recent Developments in Random Walks”, Durham University, UK, 2-7 July 2007. (*)
- 2) S. Müller: poster presentation at the conference “Recent Developments in Random Walks”, Durham University, UK, 2-7 July 2007. (*)
- 3) W. Huss: Poster presentation at the meeting on “Groups and Infinite Graphs”, ESI, Vienna, 25-27 August 2008. (*)
- 4) A. Georgakopoulos: Poster presentation at the 24th Ljubljana-Leoben Seminar, Leoben, 24-25 April 2009. (*)
- 5) A. Georgakopoulos: Poster presentation at the “Alp-Workshop”, St. Kathrein, 5-6 July 2009.

4.2.4. Conference participations without lectures or posters

- 1) W. Huss: Workshop on “Sandpile models and related fields”, EURANDOM, Eindhoven, the Netherlands, 10-13 September 2007.
- 2) J. Parkinson: Meeting on “Buildings :Interactions with Algebra and Geometry”, Oberwolfach, Germany, 20-26 January 2008. (*)
- 3) W. Woess: Member of the scientific committee, meeting on “Groups and Infinite Graphs”, ESI, Vienna, 25-27 August 2008.
- 4) A. Georgakopoulos: Symposium in Honor of Vadim Kaimanovich, Jacobs University, Bremen, Germany, 28 November 2009. (*)
- 5) A. Georgakopoulos: Conference in honour of Endre Szemerédi, Budapest, 2-7 August 2010.
- 6) W. Woess: Workshop “Graph Theory”, Mathematisches Forschungsinstitut Oberwolfach, Germany, 22 -26 February 2010.
- 7) A. Georgakopoulos: Symposium "Diskrete Mathematik" der Fachgruppe Diskrete Mathematik der DMV, ESI, Vienna, Austria, 14-15 May 2010.

List 4.3. Development of collaborations

FWF guidelines: Indication of the most important collaborations (maximum 5), that took place (initiated or continued) in collaboration please give the name of the collaboration partner (name, title, institution) and a few words about the scientific content. Please also assign one of the following **categories** to each collaboration:

N			Nature	N (national); E (European); I (other international cooperation)
↓	E		Extent	E1 low (e.g. no joint publications but mention in acknowledgements or similar); E2 medium (collaboration e.g. with occasional joint publications, exchange of materials or similar but no longer-term exchange of personnel); E3 high (extensive collaboration with mutual hosting of group members for research stays, regular joint publications etc.)
		D	Discipline	D within the discipline T transdisciplinary

N	E	D	Collaboration partner / content of the collaboration
I	E2	D	1) Name: Laurent SALOFF-COSTE Title: Prof. Institution: Cornell University Content: Brownian motion on strip complexes
	E2	D	2) Name: Sara BROFFERIO Title: Dr. Institution: Univ. Paris - Orsay Content: Internal aggregation; Brownian motion on $Sol(p,q)$
	E2	D	3) Name: Laurent BARTHOLDI Title: Prof. Institution: Univ. Göttingen Content: Spectra of horocyclic products of affine buildings
	E2	D	4) Name: Donald CARTWRIGHT Title: Reader Institution: University of Sydney Content: Affine buildings, spectral theory
	E2	D	5) Name: Vadim A. KAIMANOVICH Title: Prof. & Canadian Research Chair Institution: now Univ. of Ottawa, previously Bremen Content: boundary theory and harmonic functions

List 4.4 PhD theses / diploma theses

with an indication of the status (in progress / completed)

4.4.1. PhD Theses

The following PhD thesis is the direct outcome of the employment of W. Huss within the project.

1) Wilfried Huss: *Internal Aggregation Models*. PhD Thesis, TU Graz, completed in 2010.

The work on the following theses was not funded by the project, but the authors

profited scientifically from the strong interaction with the project as well as in the entire research group.

2) Lorenz Gilch: *Rate of Escape of Random Walks*. TU Graz, completed in 2007.

3) Ecaterina Sava: *Lamplighter Random Walks and Entropy Sensitivity of Languages*. TU Graz, completed in 2007.

4) Christoph Temmel: K-independent percolation on trees / Shearer's measure and stochastic domination. (Working title). TU Graz – Univ. Marseille, in progress.

5) Elisabetta Candellero: *Random Walks and Branching Random Walks on Free Products of Groups*. (Working title). TU Graz, in progress.

6) Tetiana Boiko: *Superharmonic functions on trees and on the hyperbolic plane* (Working title). TU Graz, in progress.

4.4.2. Diploma Theses

The work on the following theses was not funded by the project, but the authors profited scientifically from the strong interaction with the project as well as in the entire research group.

1) Elisabetta Candellero: *Branching Random Walks on Homogeneous Trees*. Univ. Torino, 2007 (prepared during an Erasmus exchange stay at TU Graz).

2) Florian Lehner: *The Line Graph of Every Locally Finite 6-Edge-Connected Graph with Finitely Many Ends is Hamiltonian*. TU Graz, completed in 2011. (Remark: this benefitted strongly from the project PostDoc A. Georgakopoulos.)

3) Larissa Stoiser: *Comparison of Quasi-Monte Carlo and Markov Chain Monte Carlo Methods*. (Working title). TU Graz, in progress.

List 4.5 Effects of the project regarding the scientific field

4.5.1. Organization of scientific events

The following international conferences were not directly funded by the project but had strong and direct scientific interaction with it.

- International Workshop on "Boundaries". June 29 - July 4, 2009, TU Graz.

Funded by EU and TU Graz. Webpage:

<http://www.math.tugraz.at/mathc/boundaries09/>

- "Alp Workshop". July 4-5, 2009, St. Kathrein am Offenegg, Austria. Funded by the FWF project of Dr. F. Sobieczky. Webpage:

<http://www.math.tu-graz.ac.at/alp-workshop/index.html>

- Short Workshop on "Analytic Enumeration Methods in Combinatorics, Probability and Number Theory". May 27-28, TU Graz. Funded by the FWF Doctoral Program (DK) "Discrete Mathematics" and by the Austrian Math. Society. Webpage:

https://www.math.tugraz.at/discrete/index.php?link=events&link2=workshop_may2010

- Spring school in “Discrete Probability, Ergodic Theory and Combinatorics”. April 4 - 15, 2011, Graz. Funded by the TU Graz, the European Science Foundation, the FWF Doctoral Program (DK) “Discrete Mathematics” and by the Austrian Math. Society. Webpage: https://www.math.tugraz.at/discrete/index.php?link=events&link2=spring_school2011

4.5.2. Information on results relevant to commercial applications

This is quite untypical for basic mathematical research.

4.5.3. Relevance of the project in the organization of the relevant scientific discipline

The project has played a major role in establishing a vibrant research group at the institute of TU Graz where it was hosted. This has favourably contributed to the standing of Mathematics within the Austrian region of Styria. See also the statement on the overall impact of this project in Section 2.3.

List 4.6. Applications for follow-up projects

with an indication of the status (submitted / approved) and the funding organization.

4.6.1. Applications for follow-up projects

FWF projects; with an indication of the project type, e.g. stand-alone project, NFN, SFB, WK, fellowship, contribution to a stand-alone publication:

1) Direct follow-up project: “*Hyperbolic Structures in Stochastics, Graph Theory, and Topology*”, project number P24028-N18, submitted in April, 2011.

The following is not a direct follow-up but parallel to the final phase of the current project.

2) FWF DK-plus “*Discrete Mathematics*”, Doctoral program W1230. W. Woess is the speaker. The program is running since May, 2010. It is mentioned here because of its importance for the present project and its follow-up.

Webpage: <https://www.math.tugraz.at/discrete/>

4.6.2 Applications for follow-up projects

(International projects; eg. ERA project, ESF)

1) DFG (Deutsche Forschungsgemeinschaft) individual fellowship of Sebastian MÜLLER at the same host institute, subsequent to his collaboration within the present project, in the period 1/10/2007 – 30/9/2009.

2) Subsequently, MÜLLER obtained a Marie Curie fellowship at Univ. Marseille (recently concluded).

4.6.3 Other projects

There were, resp. are other national and international projects which were, resp are strongly linked scientifically with the present one, although not to be considered as “follow-up”.

1) DocForte PhD grant (Austrian Academy of Sciences) of Elisabetta CANDELLERO, since 01/04/2010.

2) ESF (European Science Foundation) research network “RGLIS - Random Geometry of Large Interacting Systems and Statistical Physics“, since June 2010. W. Woess is a member of the steering committee.

5. Zusammenarbeit mit dem FWF

Bewertung von Aspekten der Zusammenarbeit mit dem FWF:

Skala

- 2 sehr unzufriedenstellend,
- 1 unzufriedenstellend;
- 0 angemessen;
- +1 zufriedenstellend;
- +2 sehr zufriedenstellend.
- X nicht beansprucht

Regelwerk

(Richtlinien für Programm, Antrag, Verwendung, Bericht)

Wertung

Antragsrichtlinien	Umfang	1
	Übersichtlichkeit	1
	Verständlichkeit	1

Verfahren (Einreichung, Begutachtung, Entscheidung)

	Beratung	1
	Dauer des Verfahrens	1
	Transparenz	2

Projektbegleitung

Beratung	Verfügbarkeit	1
	Ausführlichkeit	1
	Verständlichkeit	2

Durchführung Finanzverkehr (Überweisungen, Gerätebeschaffungen, Personalwesen)		2
--	--	----------

Berichtswesen/ Prüfung/ Verwertung

	Aufwand	0
	Transparenz	2
	Unterstützung bei Öffentlichkeitsarbeit/ Verwertung	x

Anmerkung zur Zusammenarbeit mit dem FWF:

Insgesamt ist meine Meinung sehr positiv, was Transparenz, Abwicklung und wissenschaftliche Zuverlässigkeit des FWF betrifft (insbesondere im Vergleich zu den sehr nebulösen, undurchsichtigen Kriterien und der überbordenden Bürokratie bei EU-Projekten).

Graz, 25 August 2011
Wolfgang Woess