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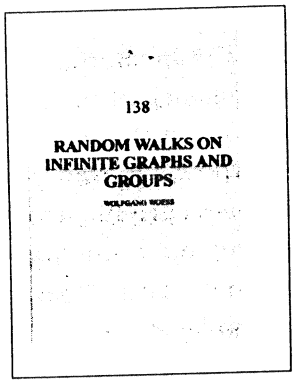


W. Woess

Random walks on infinite graphs and groups

(Cambridge tracts in mathematics; 138)
Cambridge: Cambridge university press, 2000
334 p., prijs £40,-
ISBN 0-521-55292-3

This book is a research monograph dealing with discrete time random walks on countable graphs. In the first chapter, the author presents selected topics around what he calls the 'type problem', meaning the determination of the recurrent or transient nature of such walks on different graphs. He also succeeds in slightly annoying the reviewer, who was involved in the first proofs of many of these results as part of the Rennes group KGB in the 1970's, by not presenting any of these results or even giving the references to the Asterisque volume and other French literature where they appeared. On the other side, too much emphasis is placed in the reviewer's opinion on the contributions of Varopolous and the Italian school. The second chapter deals with the determination of the spectral radius associated with random walks, the computation of Green functions, and associated isoperimetric inequalities. In chapter three, the asymptotic behaviour of transition probabilities is discussed through the presentation of diverse local limit theorems, and the final chapter contains a brief excursion into topological boundary theory, which allows identification of the harmonic functions associated with random walks.

So far the content. The choice of material, given the length of the book, is necessarily somewhat incomplete and not unbiased. Moreover, the organization of the material seems haphazard and without a clear line — for instance, chapter two begins immediately with technicalities leaving the motivation completely to the reader. One clearly gets the impression that the author has 'learned through writing', as he states in the introduction. More seriously, many of the interesting problems in this area today, such as recurrence and reinforcement, scaling behaviour and criticality, and in general the strong interaction with problem areas in statistical physics, go without mention. This, together with the willy-nilly organization, produces the curious effect of making an exciting subject dull. However, the subjects that are covered are also of interest, and in these areas this book can serve as a learning tool and a reference.

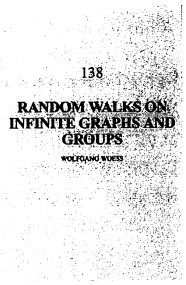
M. Keane

Author's reply to a review by M. Keane

Cambridge University Press has forwarded to me a copy of the review by Prof. M. Keane, in NAW 5/4 no. 1 (2003), of my monograph *Random Walks on Infinite Graphs and Groups*, Cambridge Tracts in Mathematics 138, 2000.

The tone of the review is very unusual, if not personally offensive — not only in my own view; I also consulted several colleagues before deciding to respond.

Let me start with Prof. Keane's criticism regarding the choice of the material. As stated clearly in the preface, the topic of the book is the interplay between the behaviour of time-homogeneous Markov chains and the geometric-algebraic-combinatorial structure of the underlying state space. This is currently a rather active field of research, and I have the impression that my book has been well received by the scientific community, as one may also see from other reviews like the ones in Bull. London Math. Soc. 33 (2001) p. 243–245, Acta Sci. Math. (Szeged) 67 Vol. 1–2 (2001), or Bull. Amer. Math. Soc. 39 (2002) p. 281–285. The 'missing' topics indicated by M. Keane would correspond to a very different general theme; there are indeed very different ones among the "many of the interesting problems in this area today". (Quotation marks refer to prof. Keane's review.) Also, I do not believe that there is "too much emphasis [...] on the contributions of Varopoulos", since Varopoulos not only has solved a longstanding open problem (classification of finitely generated, recurrent groups) but also opened the door to a wealth of new methods — notably analytic and not probabilistic in flavour — for examining the asymptotics of transition probabilities on graphs. Regarding the "Italian school", I guess that Prof. Keane refers to myself and to close colleagues of mine — I think that it was my right to dedicate a modest part of my monograph to the achievements of those people, in particular including myself.



The most irritating piece of the review is the phrase "One clearly gets the impression that the author has 'learned through writing', as he states in the introduction." The method applied by the reviewer is to quote in such a way as to obtain the opposite of the original meaning; here, Prof. Keane seems to suggest that I did not know what I was going to write when I started to work on my book. Let me explain the true meaning of the respective phrase of mine. Most of us know a good amount of the literature in the field in which we are working (and I believe to have a rather wide overview) to the extent that we know the results and the basic ideas of many other researchers. But there is only a comparatively small portion of papers where we really went through every detail, as required for elaborating the results in such a way that they can for example be presented in a monograph. Here, I have elaborated results from at least eighty original papers in detail,

often giving shorter, different, or new proofs — this has been my own fruitful 'learning'.

The above-mentioned allusion together with the statements that the "organization of the material seems haphazard and without a clear line" and "willy-nilly organization" are in contrast not only with the opinion of other reviewers, but also with the story how this book was prepared. First, I had published a long survey article with the same title as the book, that appeared in Bull. London Math. Soc. 26 (1994) p. 1–60. It was organized in a similar way as the book; each chapter is centered around one specific theme, starting with general results and then passing to specific structures. The range of topics is somewhat wider than the selection in the book; in particular, the volume by Guivarc'h, Keane and Roynette is cited there. My impression is that this survey was warmly welcomed by the scientific community; it has been cited very frequently. The letter from the editor, in accepting it for publication, started with the phrase "Thanks very much for your excellent [...] survey." In the sequel, one of the editors of Cambridge Tracts in Mathematics suggested to me to write a book on the same topic. I submitted a detailed proposal, which was accepted by an anonimous referee. Upon completion of each single chapter, I sent the latter to various colleagues working in the field, asking for comments, criticism, suggestions, et cetera, which were taken into account in the final version.

There are of course two points of legitimate criticism in Prof. Keane's review. One is the lack of more detailed motivation at the beginning of Chapter II, and the other is the missing citation of the influential book by Guivarc'h, Keane and Roynette, *Marches Aléatoires sur les Groupes de Lie*, Lecture Notes in Math. 624, Springer, 1977. I did of course cite the latter not only in my survey, but also in various of my research papers. It was not cited in my book basically because the latter deals exclusively with discrete structures, and none of those results on Lie groups is used. However, it certainly would have been wise to include the reference in the "Notes and Remarks" at the end of Chapter I.

Wolfgang Woess